Phase unwrapping

Rüdiger Gens

Outline

- Basics
- Factors influencing the phase
- Terminology
- Phase filtering
- Phase unwrapping algorithms
  - Path-following methods
  - Minimum-norm methods
- Weighting factors
- Trends and challenges

Importance of phase

- transmission or reception of <u>coherent</u> signals
- coherent processing
  - synthetic aperture radar (SAR)
  - synthetic aperture sonar
  - seismic processing
  - adaptive optics
  - magnetic resonance imaging (MRI)
  - aperture synthesis radio astronomy
  - optical and microwave interferometry

Relation to physical quantity

- in many applications the phase relates to a physical quantity
  - adaptive optics $\rightarrow$ wavefront distortion
  - MRI $\rightarrow$ degree of magnetic field inhomogeneity in the water/fat separation problem
  - astronomical imaging $\rightarrow$ relationship between the object phase and its bispectrum phase
  - interferometry $\rightarrow$ surface topography

Optical and SAR interferometry

- Optical interferometry
  - coherent signal source: laser
  - application: holography
- SAR interferometry
  - coherent signal source: synthetic aperture radar
  - primary application: digital elevation models

Focus on phase unwrapping in SAR interferometry

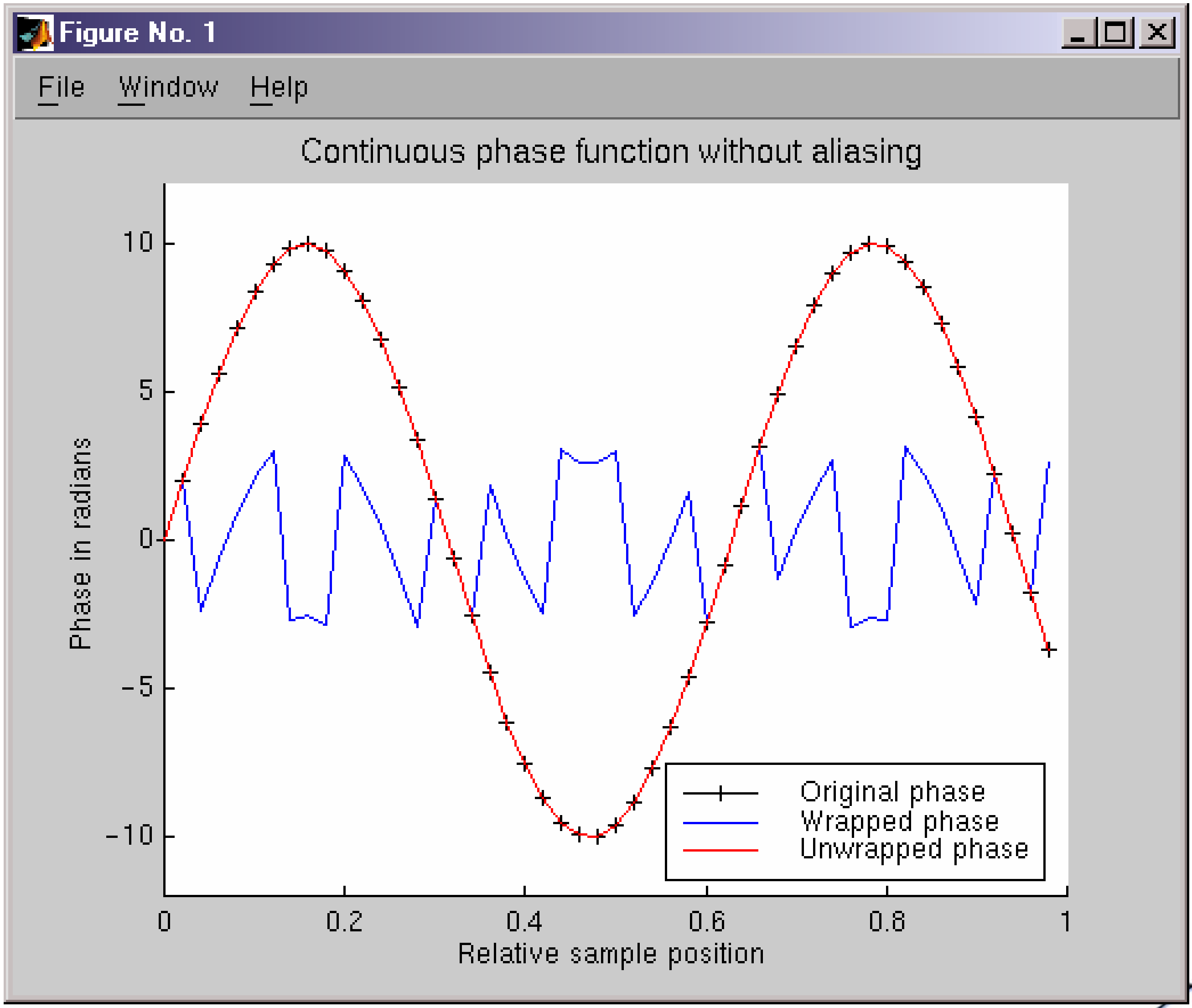
Why phase unwrapping?

- continuous phase information is sampled in a discrete wrapped phase
- looking for the correct integer number of phase cycles that needs to be added to each phase measurement to obtain the correct slant range distance
- absolute phase is wrapped into the interval $(-\pi,+\pi] \rightarrow$ ambiguity problem
- solving ambiguity referred to as phase unwrapping

Factors influencing the phase

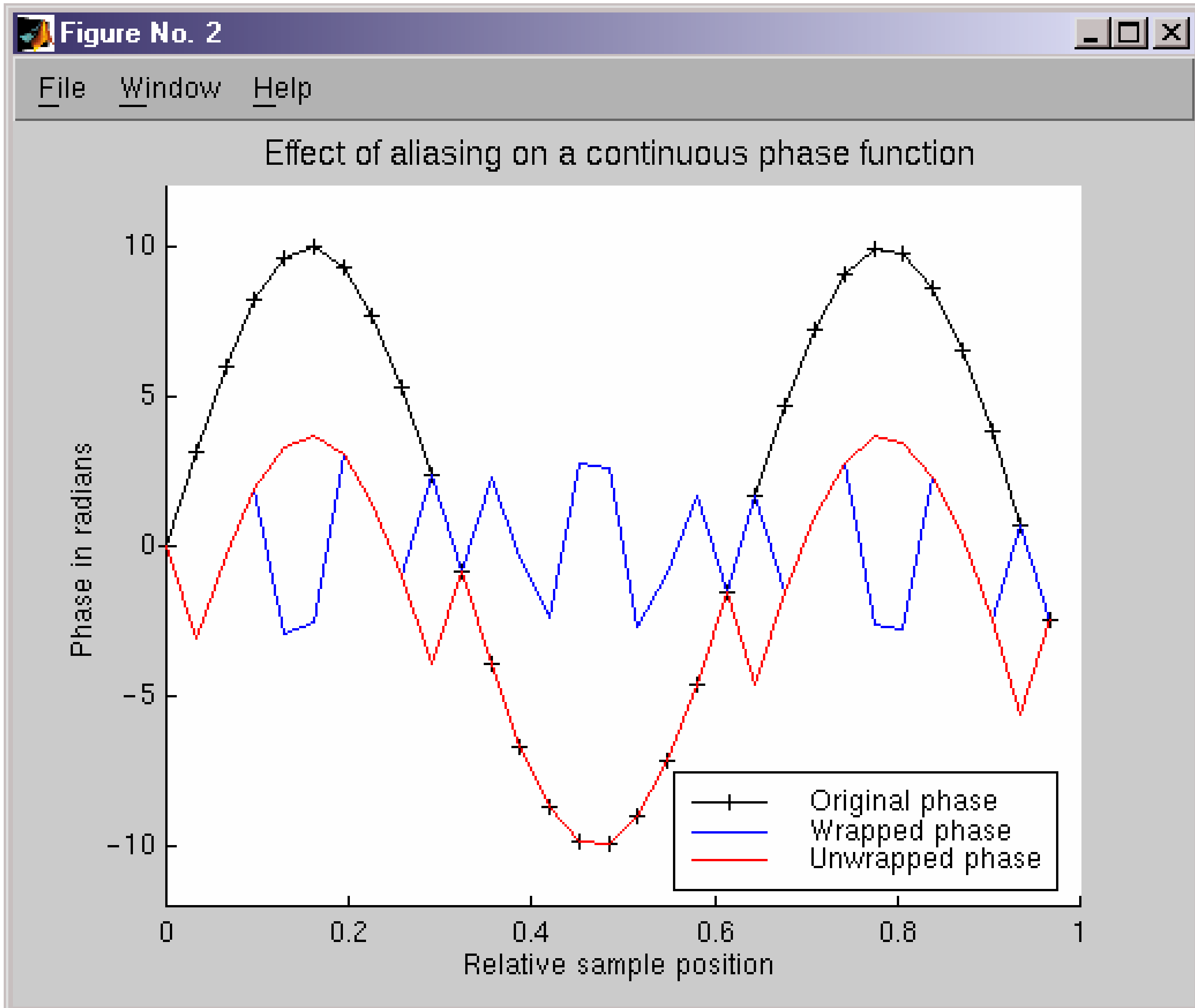
- phase aliasing → insufficient sampling rate
- phase noise
- thermal noise → sensor electronics
- temporal change → different backscatter
- baseline geometry → fringe density

Influence of phase aliasing

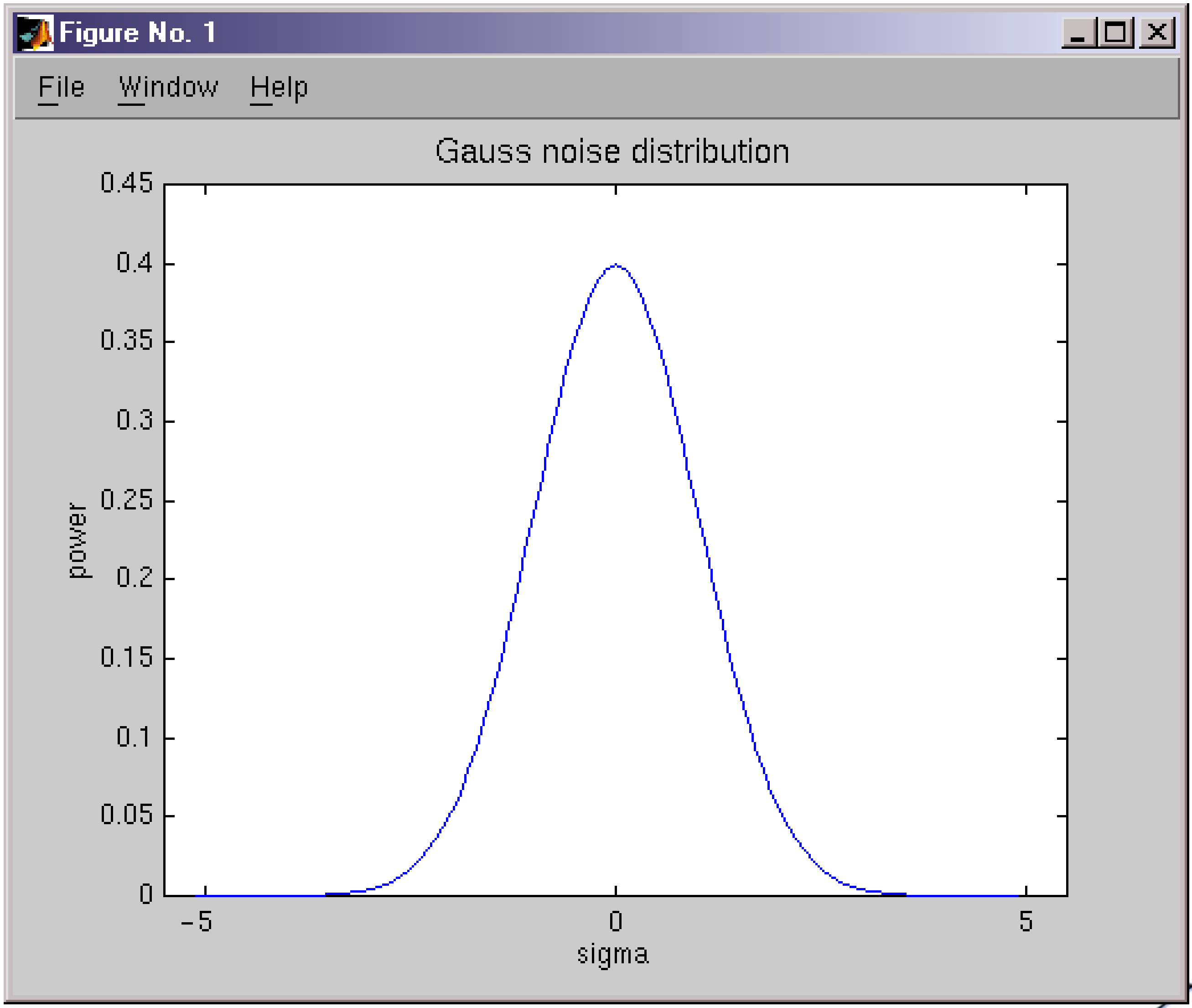


Influence of phase aliasing

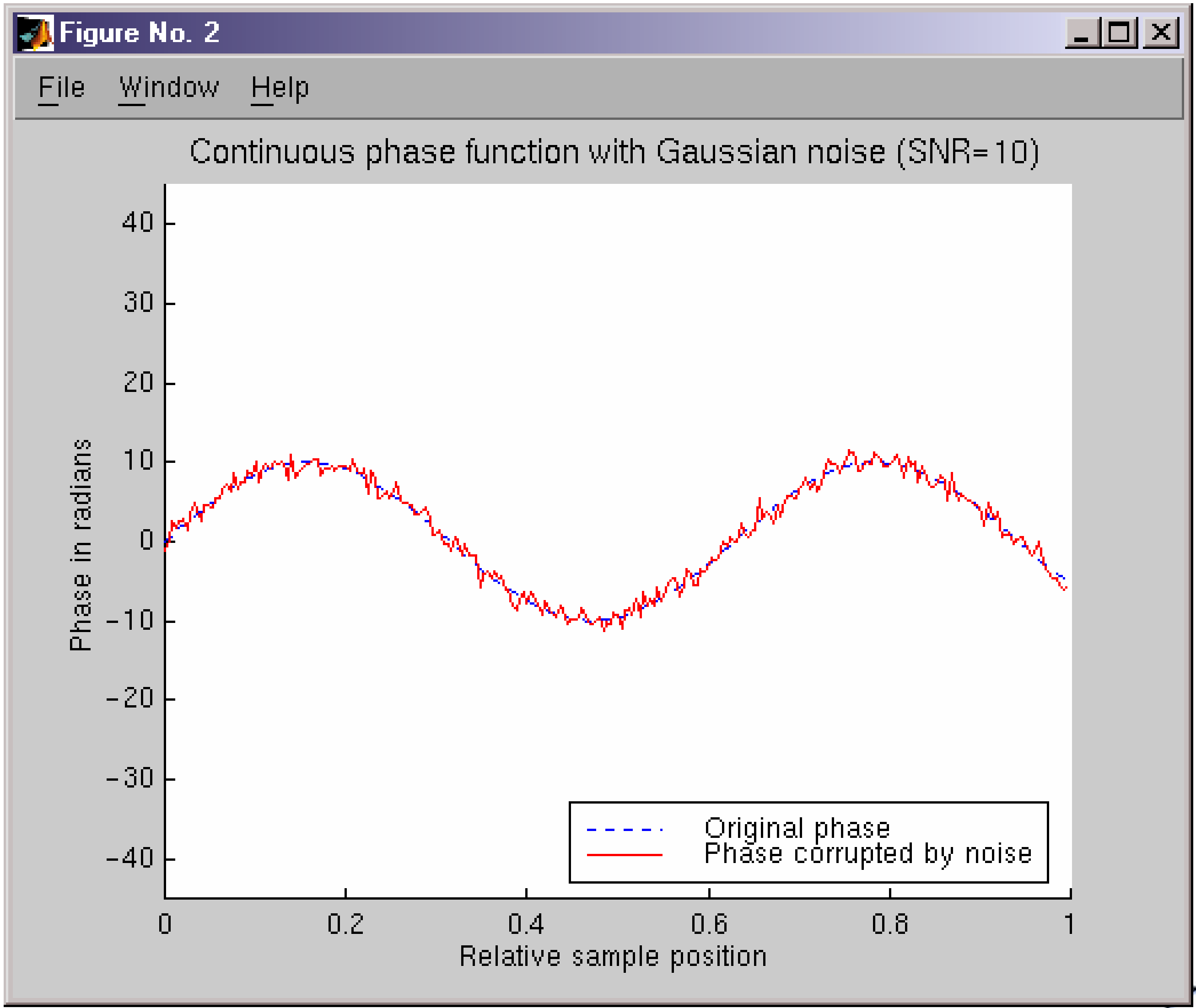
Phase unwrapping



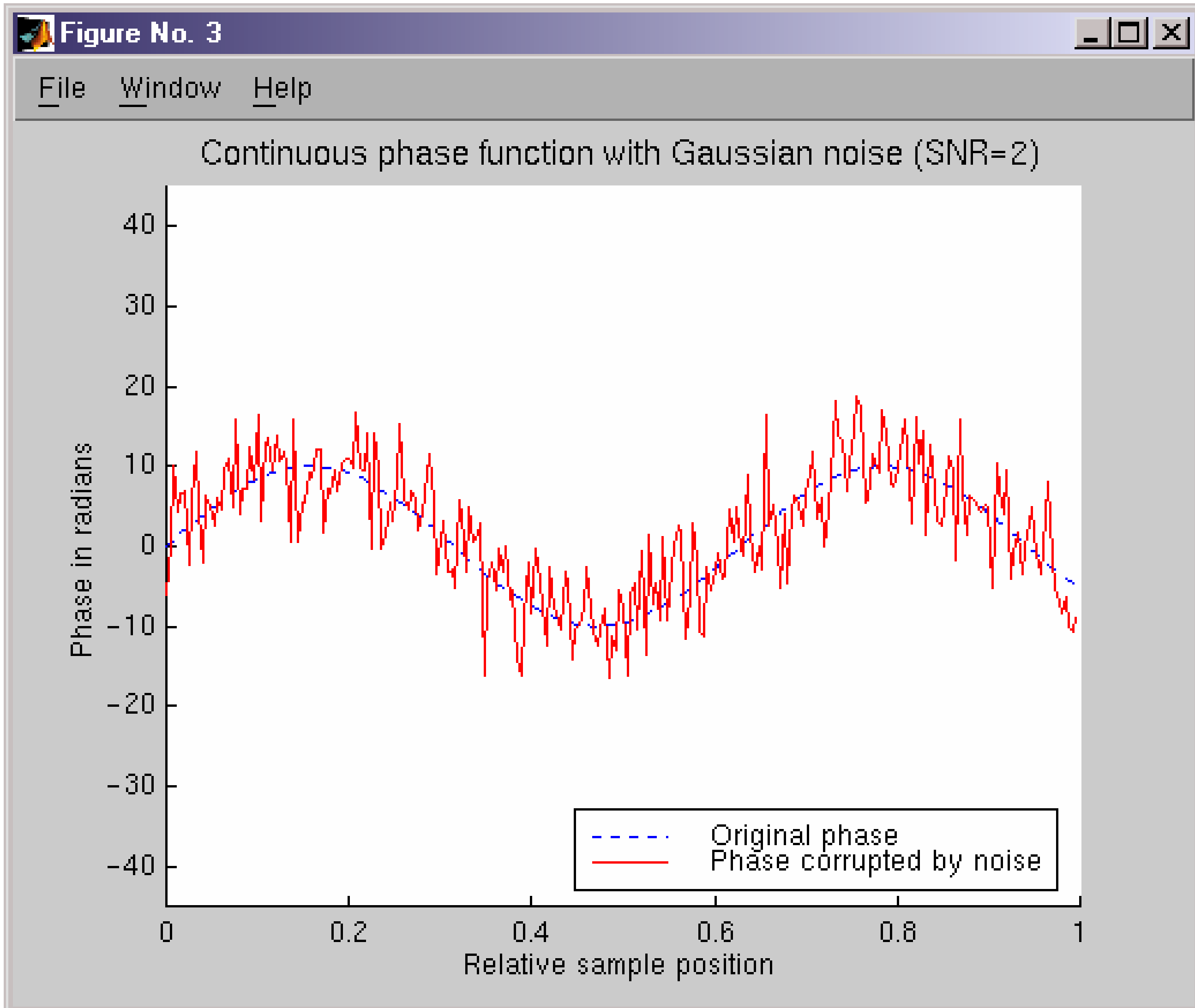
Influence of phase noise



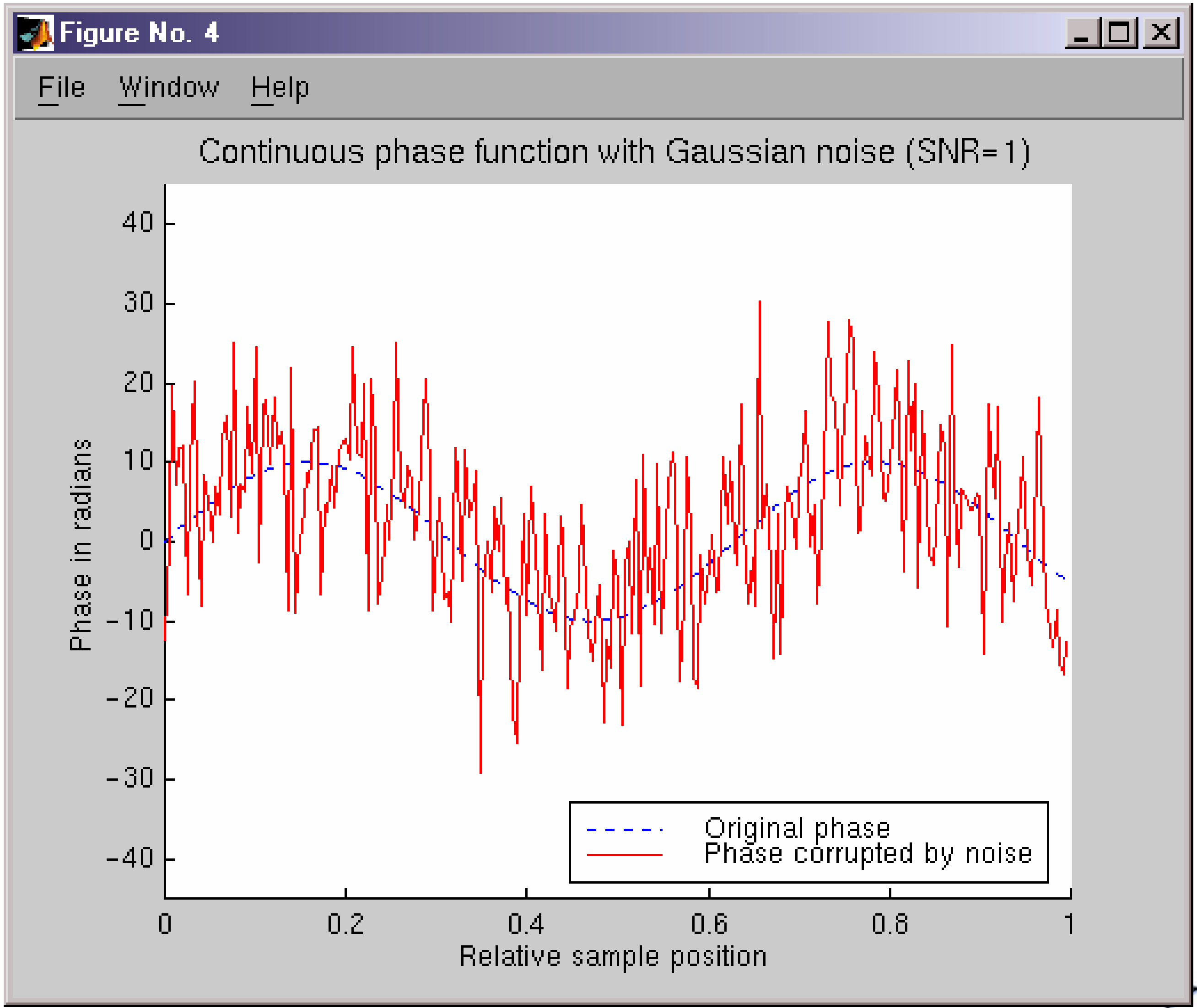
Influence of phase noise



Influence of phase noise



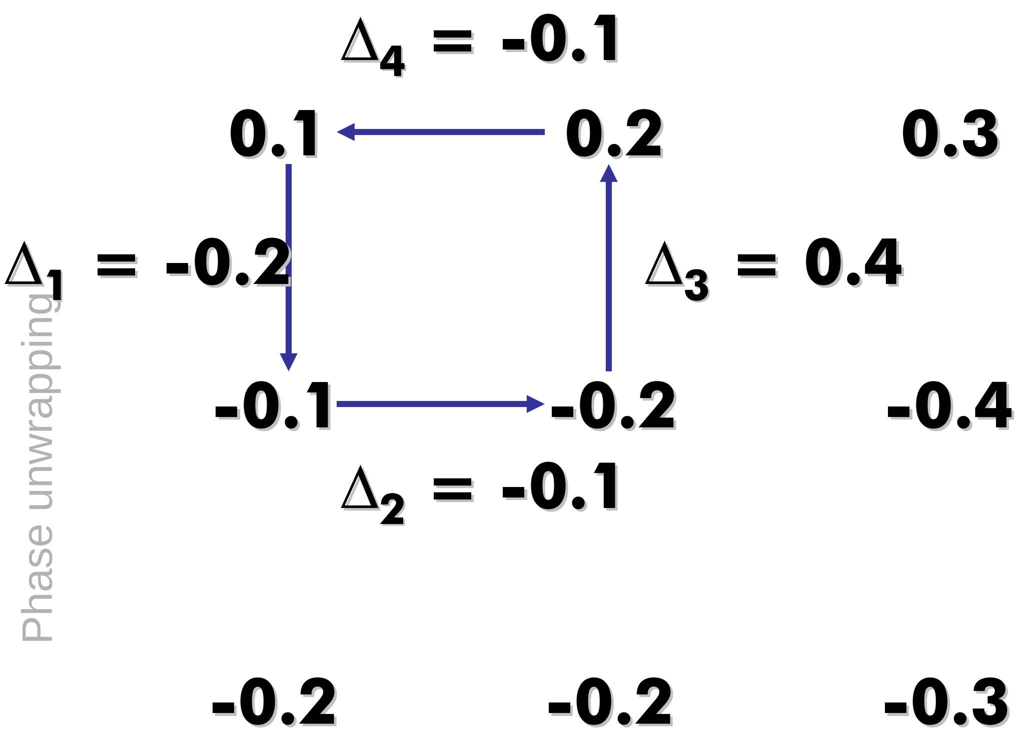
Influence of phase noise



Phase unwrapping terminology

- phase gradient
- phase discontinuity
- residue
- polarity
- charge
- branch cut

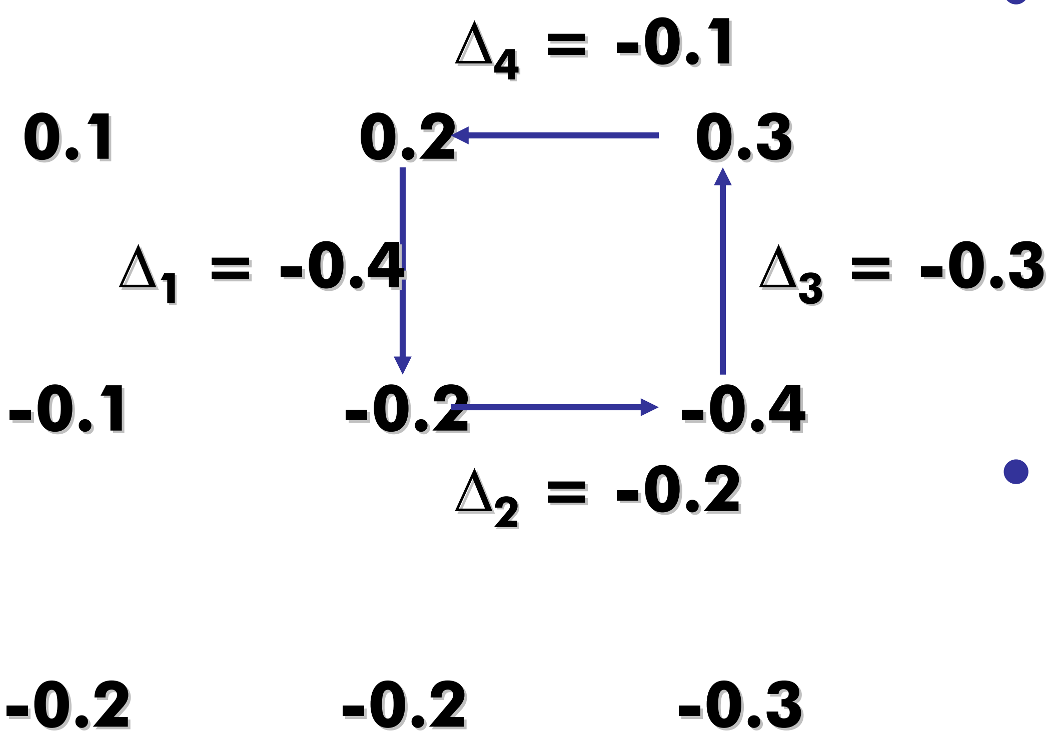
Phase gradients



- small portion of wrapped phase image
- values divided by 2π
- phase gradients defined as phase difference of adjacent pixels

$$q = \sum_{i=1}^{4} \Delta_i = 0$$

Inconsistencies and residues



- integrating wrapped phase gradients around every 2x2 sample path in the entire image
- residue (discontinuity) if sum of phase gradients not zero

$$q = \sum_{i=1}^{4} \Delta_i = -1$$

Polarities and charges

0.1		**0.2**		**0.3**
	0		-	
-0.1		**-0.2**		**-0.4**
	0		0	
-0.2		**-0.2**		**-0.3**

- non-zero integrals define residues
- sign of the residues define polarity or charge of a residue

Branch cuts

- connection of residues with opposite polarity are referred to as branch cuts
- prevent any integration path from crossing the branch cuts
- residues and branch cuts are essential part of path-following phase unwrapping methods

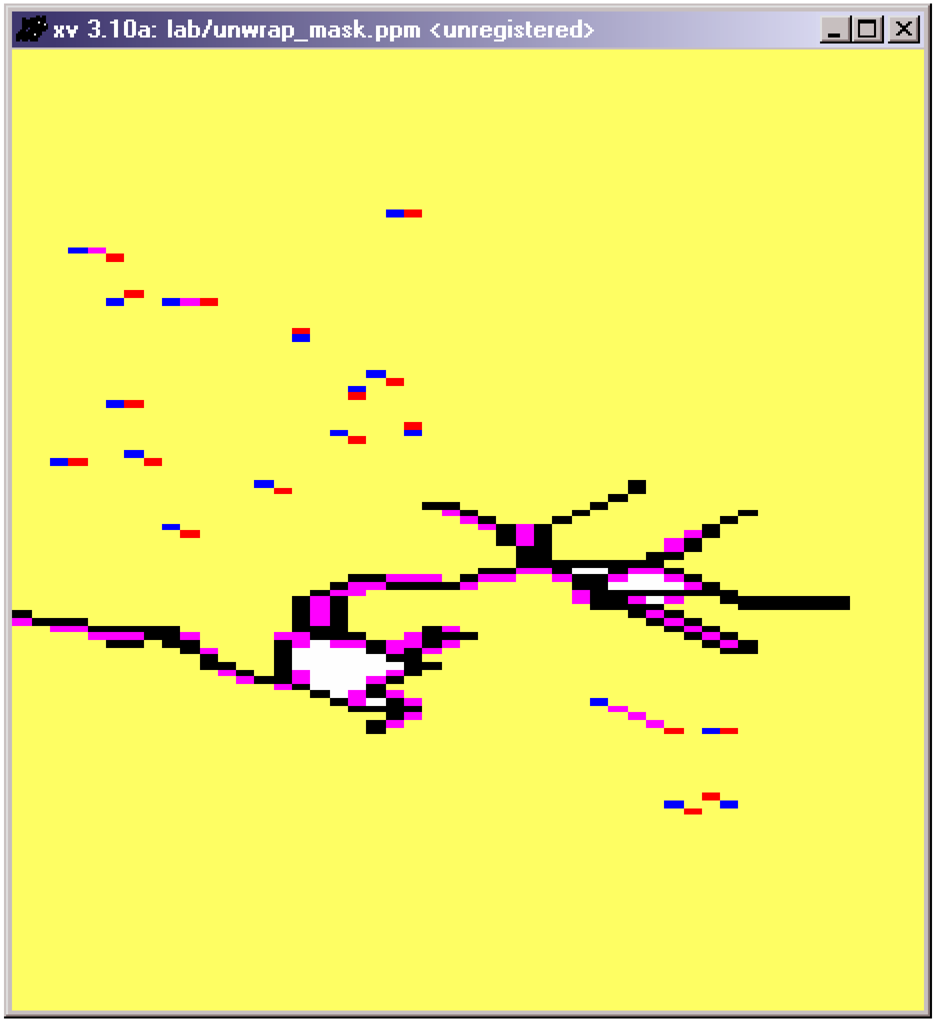
Ideal phase unwrapping case

- no residues (discontinuities) in the images
- integration of the phase gradients over the whole data set
- integration independent from integration path

Phase unwrapping reality

- phase noise
- phase discontinuities resulting in residues
- high fringe rates in foreshortening and layover regions $\rightarrow$ fringes cannot be separated
- shadow regions
  $\rightarrow$ no phase unwrapping possible at all
- integration not independent from its integration path

Phase unwrapping reality



- white:non-integrated
- black: grounding
- purple: branch cut
- red: neg. residue
- blue: pos. residue
- yellow: integrated

Phase filtering

- interferogram power spectra
  - "white" component generated by thermal noise and loss of coherence
  - narrow band component related to fringes
- fringe rate determined by
  - look angle
  - along-track changes in the baseline
  - any motion of the scene along the line of sight

Phase filtering

- approach developed by Goldstein and Werner
- adaptive filtering sensitive to
  - local phase noise
  - fringe rate
- segmentation of interferogram into overlapping rectangular patches

Phase filtering

- estimation of the power spectrum
  - computing by smoothing the intensity of the two-dimensional FFT
- spatial resolution of the filter adapts to the local phase variation
  - regions of smooth phase are strongly filtered
  - regions with high phase variance are weakly filtered

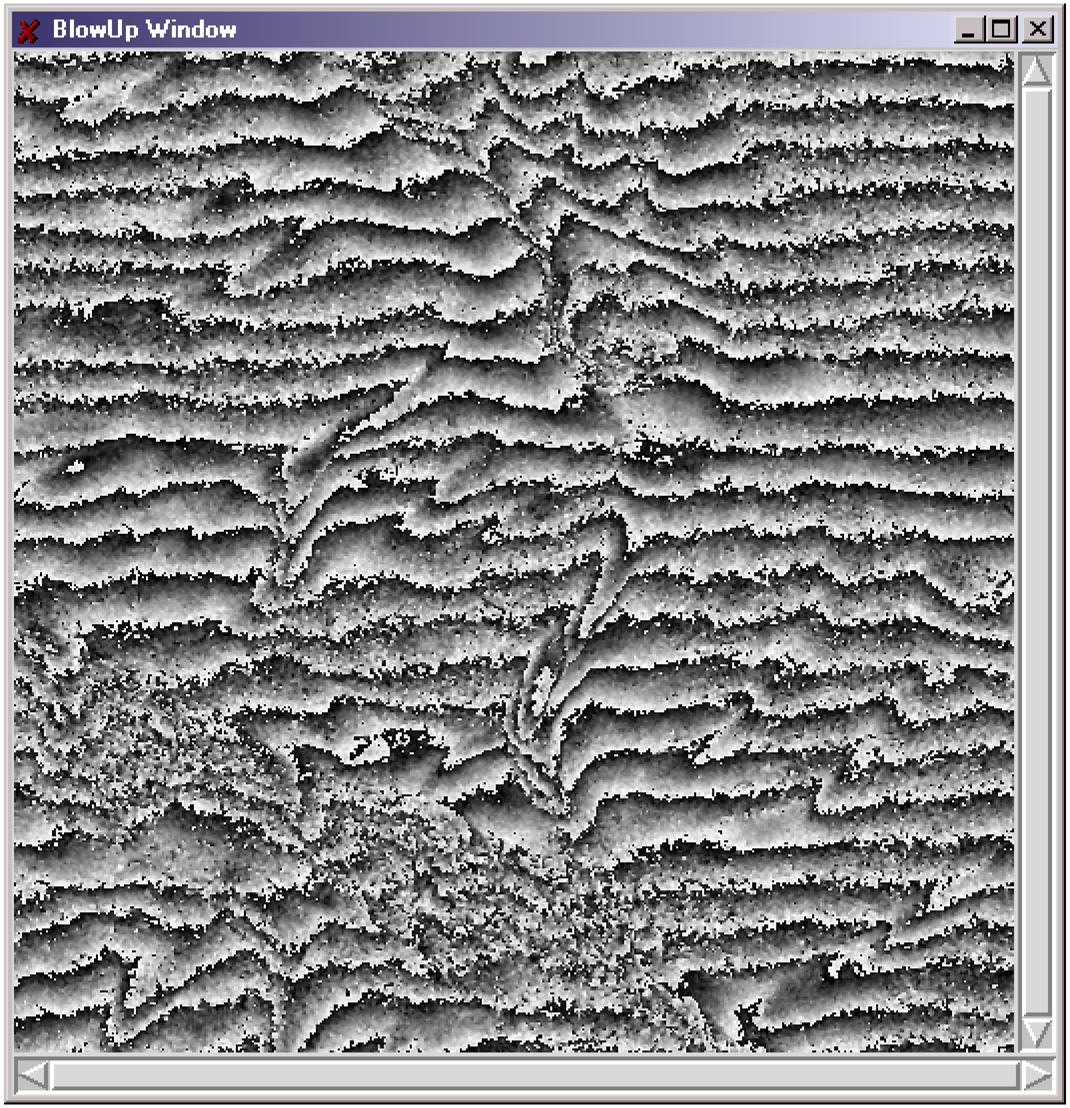
Phase filtering

$$H(u,v) = \left| Z(u,v) \right|^{\alpha}$$

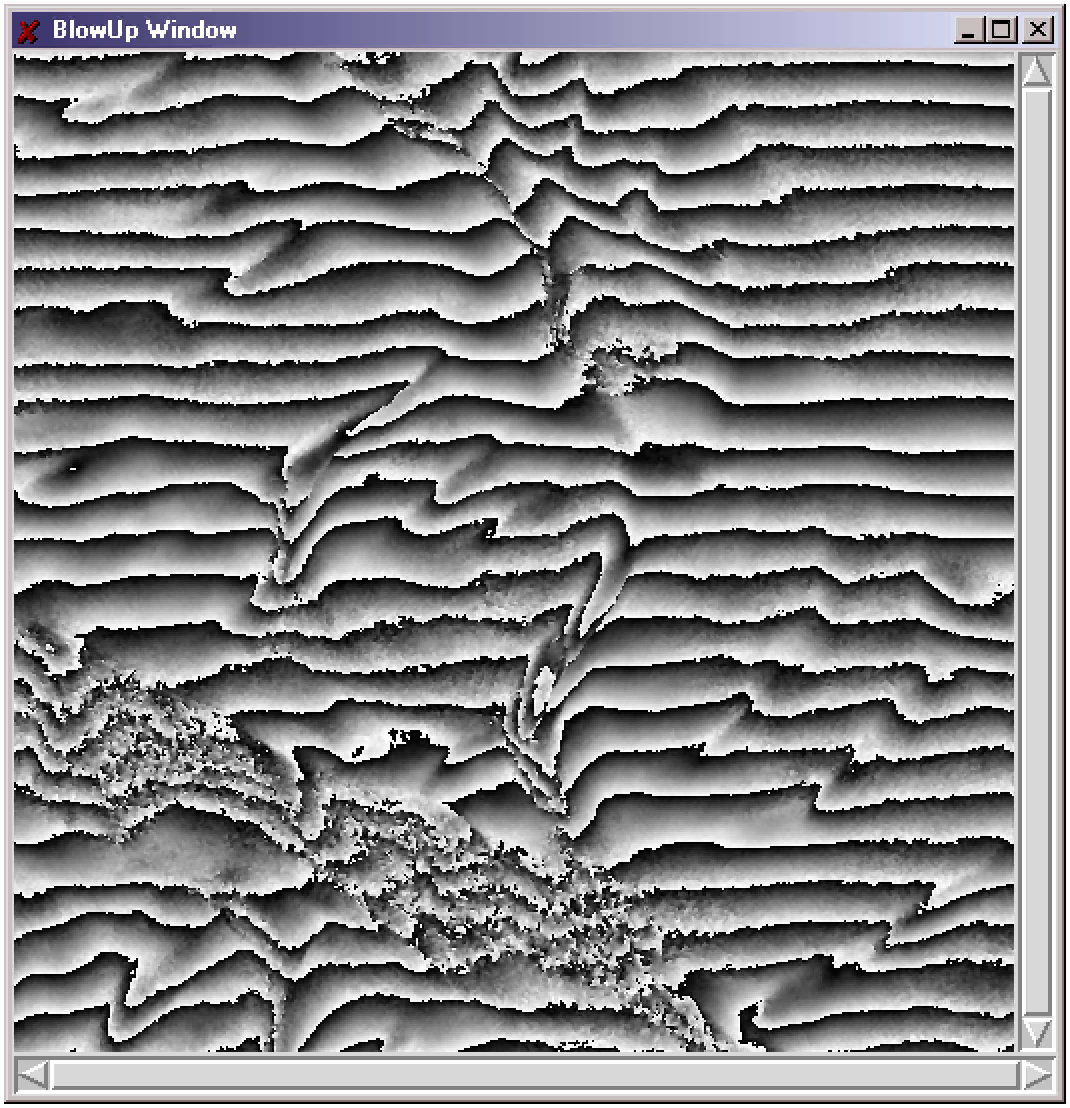
$$Z(u,v) = \exp\left\{ -\frac{\frac{u^2}{\sigma_u^2} - \frac{2uv}{\sigma_u \sigma_v} + \frac{v^2}{\sigma_v^2}}{2(1-\rho^2)} \right\}$$

- ***H(u,v)*: adaptive filter**
- ***Z(u,v)*: power spectrum**

Unfiltered phase



Filtered phase



Phase unwrapping algorithms

- no standard procedure to solve the phase unwrapping problem
- large variety of algorithms developed
- generally trade off between accuracy of solution and computational requirements
- two types of strategy to solve the phase unwrapping problem
  - path-following methods
  - minimum-norm methods

Path-following methods

- local approach
- Goldstein`s branch cut algorithm
- Flynn`s minimum discontinuity algorithm
- minimum cost flow (MCF) networks
- minimum spanning tree algorithm

Goldstein`s branch cut algorithm

- classical path-following method
- defines branch cuts between all detected residues
- algorithm prevents any integration path from crossing these cuts
- residues need to be balanced
  - connection with a residue of opposite polarity
  - connection with the image border

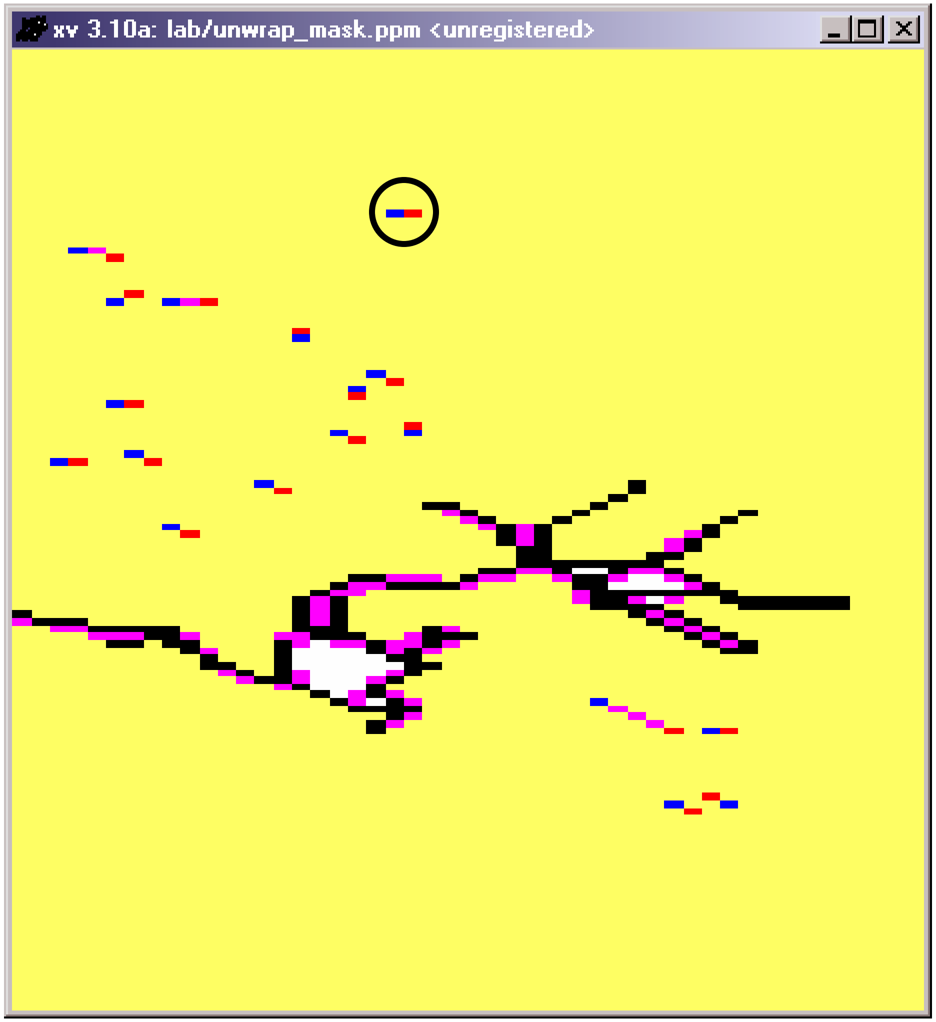
Goldstein`s branch cut algorithm

- approach minimizes the sum of the branch cut length
- algorithm
  - is computationally very fast
  - requires little memory
- lack of weighting factors that could be used for guiding the placement of branch cuts
  $\rightarrow$ poor performance in areas of low coherence

Goldstein`s branch cut algorithm

- algorithm tends to create isolated regions by closed branch cuts

Goldstein`s branch cut algorithm



- several enhancements suggested
  - removal of so-called dipoles
  - phase filtering reduces number of residues

  → higher fringe visibility

  → reduced phase noise

Flynn`s minimum discontinuity algorithm

- finds a solution that actually minimizes the discontinuities
- high memory and computational requirements
- tree-growing approach
  - traces paths of discontinuity in the phase
  - detects paths that form loops
  - minimizes the discontinuities by adding multiple of 2π to the phase values enclosed by the loops
- works with or without weighting factors

Minimum cost flow networks

- formulates the phase unwrapping problem as global minimization problem with integer variables
  - uses the fact that phase differences of neighboring pixels can be estimated with a potential error that is an integer multiple of 2π
- optimization using MCF networks provides position of branch cuts
- definition of costs assigned to flows within network includes weighting factors in the process

Minimum cost flow networks

- relatively new approach
- uses general purpose software packages
  - MCF networks widely available
  - large field of research in itself
- designing MCF networks more adapted to the specific constraints of phase unwrapping still a major research issue

Minimum spanning tree algorithm

- adaptation of Goldstein`s algorithm
- approximates a minimum Steiner tree
- builds a single tree containing all charges
  - drawing branch cuts to next nearest charge to the tree when charge of current tree becomes neutral
- definition of weights on phase gradients
  - searching for the next charge to the tree with Dijkstra`s shortest path algorithm

Minimum spanning tree algorithm

- cuts are associated with the phase differences
  - guarantees that the tree does not close on itself

Minimum-norm methods

- global approach

- least-squares phase unwrapping
- minimum L_p-norm phase unwrapping

Least-squares phase unwrapping

- solution of phase unwrapping by discretized partial differential equations (PDEs)
- least-squares favorable for solution of PDEs
  - solution leads to a linear equation
  - $\rightarrow$ integrates the residues to minimize the gradient differences
- works in weighted and unweighted form

Least-squares phase unwrapping

- unweighted least-square problem described as discretized Poisson equation that can be solved by
  - Fast Fourier Transformations (FFTs)
  - discrete cosine transforms (DCTs)
  - unweighted multigrid method
- weighted least-squares approach requires iterative methods
  - Picard iteration method
  - preconditioned conjugate gradient (PCG) method
  - weighted multigrid method

Minimum L_p-norm phase unwrapping

- generalization of weighted least-squares approach
- requires solution of a non-linear PDE implemented in an iterative scheme
- double iterative structure makes algorithm computationally very intensive
- generates data dependent weights (optional)

Weighting factors

- important feature for a large number of algorithms for their improved performance
  - also referred to as quality maps
- define the quality of phase data on pixel level
- increasing interest with the introduction of minimum cost flow networks
- various sources
- number of combinations countless

Sources for weighting factors

- correlation coefficient (coherence)
  - enhanced and re-scaled
- pseudo-correlation
  - correlation with uniform magnitude
- phase derivative variance
  - local sample variance of the partial derivatives of the phase data
- maximum phase gradient
  - magnitude of the largest phase gradient

Sources for weighting factors

- residue density
- flatness of unwrapped phase
- smoothness of unwrapped phase
  - sum of absolute values of the phase gradient
- statistically derived values
- masks used for excluding data from phase unwrapping process

Trends and challenges

- complexity of many approaches increases the demand in memory and computational efficiency
- improved hardware performance compensated by size of data sets used
- results of shuttle radar topography mission (SRTM) could improve phase unwrapping results
- dealing with large volume data requires the independence of human interaction