



# Height systems

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# Why bother about height systems?

- give a meaning to a value defined for height
- combination of measurements from different sources
  - GPS measurements vs. leveling measurements
- three-dimensional calculations
  - SAR interferometry

Height systems



# Relevant terms

- spheroid
  - any surface resembling a sphere
  - an ellipsoid of revolution
- ellipsoid
  - defined by axes, flattening and eccentricity
- flattening and eccentricity
  - characterize the deviation from a sphere

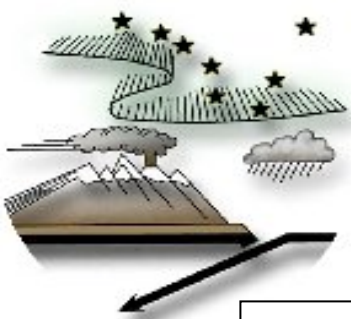
Height systems



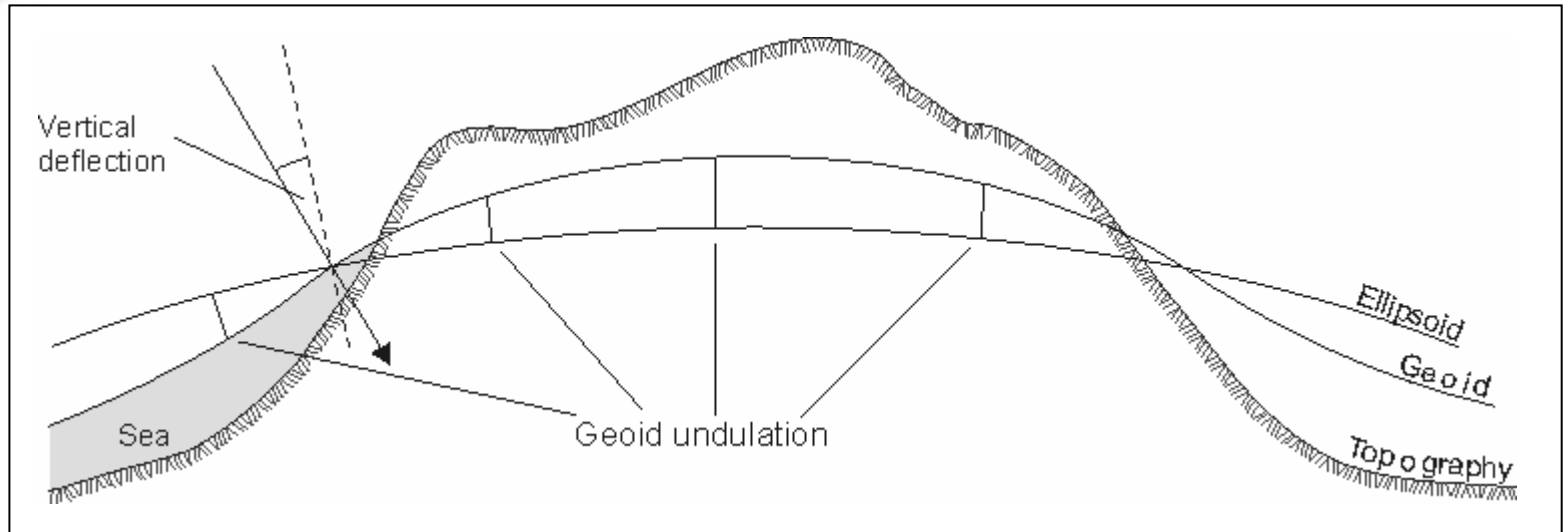
# Approximation vs. Reality

- ellipsoid is a good approximation to the shape of the Earth but not an exact representation
- Earth surface is everywhere perpendicular to the direction of gravity  
→ *equipotential surface*
- true shape of the Earth is known as *geoid*

Height systems

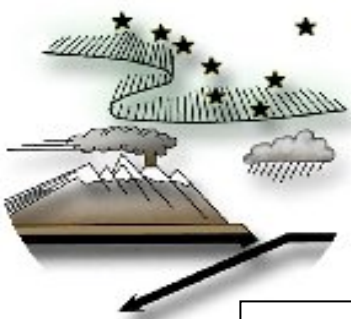


# Reference surfaces

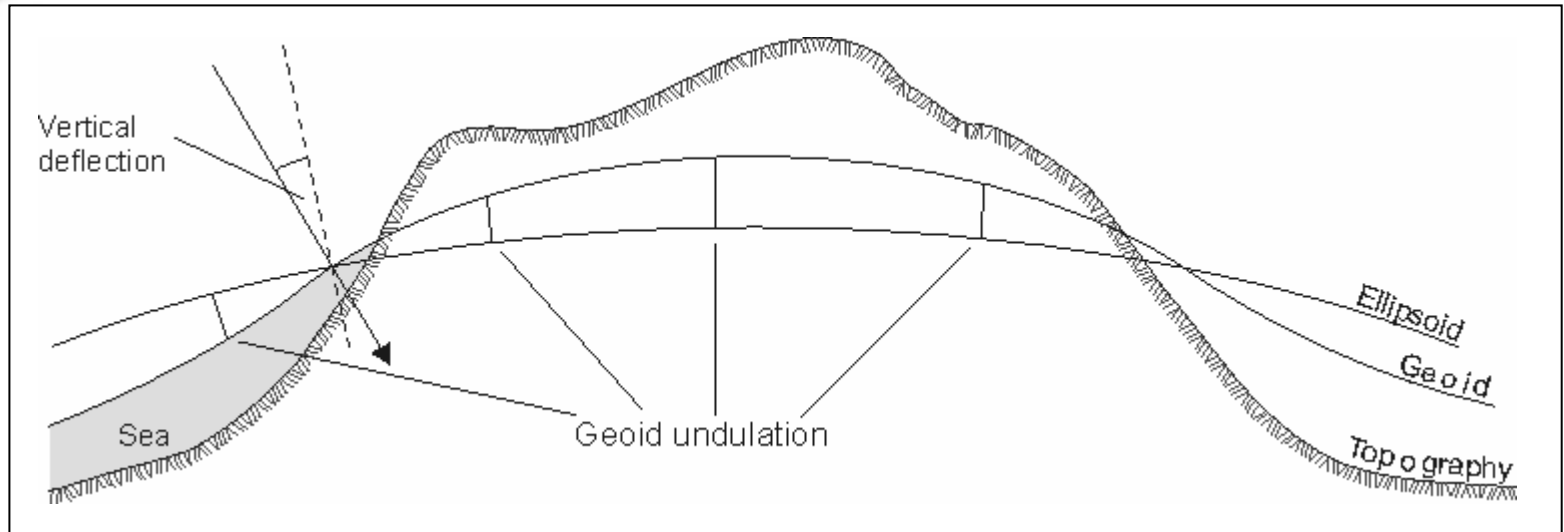


Height systems

- *geoid undulation*: vertical separation between geoid and reference ellipsoid
  - differences between  $\pm 100$  m
  - global root mean square of around 30 m



# Reference surfaces



Height systems

- *vertical deflection*: angle between the ellipsoid normal and the plumb line
  - usually resolved in a north-south component  $\xi$  and an east-west component  $\eta$
  - angles usually amount to a few arc seconds



# Global earth model

- geoid defined by a set of coefficients of a spherical harmonic expansion  
→ global earth model
- several models available
  - OSU91
  - Earth Geopotential Model 1996 (EGM96)

Height systems



# Geopotential number

- different height systems can be related to each other by the geopotential number  $C$

$$C = W_0 - W = \int_{\text{geoid}}^{\text{point}} g \, dn$$

- $W$  and  $W_0$ : the potentials of gravity of a point and the geoid
- $g$ : gravity value
- $dn$ : leveling increment

Height systems





# Geopotential number

- different heights calculated by dividing the geopotential number by a gravity value

Height systems



# Heights

- dynamic height
  - constant normal gravity  $\gamma_0$  for an arbitrary standard latitude (usually 45 degrees)
  - no geometrical meaning
- orthometric height
  - natural “height above sea level”
  - measured along the current plumb line from the foot point on the geoid and the point on the surface
  - gravity value: mean gravity

Height systems



# Heights

- normal height
  - vertical distance from terrain surface to the ellipsoid reduced by the height anomaly
  - measured along the ellipsoidal normal
  - gravity value: mean normal gravity

Height systems



# Solution

- ellipsoid is convenient reference frame
  - mathematical figure
  - provides good approximation to the geoid
- geoid better height reference system
  - reference to mean sea level allows to use tide gauges as height reference points
  - physical significance: ensures horizontal representation of water surfaces such lakes and seas

Height systems