Calculation of ASF CEOS Metadata Values

Manual

by

Tom Logan and Rüdiger Gens

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TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

LIST OF SYMBOLS

α	Squint angle
γ	Difference from geodetic to geocentric latitude ($\gamma = \theta_D - \theta_C$)
Δ_{AZ}	Azimuth Line Spacing
Δ_{GR}	Ground Range Pixel Spacing
ΔL	
Δ_{SR}	Slant Range Pixel Spacing
$\Delta\theta$	Look angle update
$\Delta\varphi$	Change in incidence angle
θ	Look angle
θ_C	Geocentric Latitude
θ_D	Geodetic Latitude
θ_{SC}	Platform Latitude
λ	Radar Wavelength (meters)

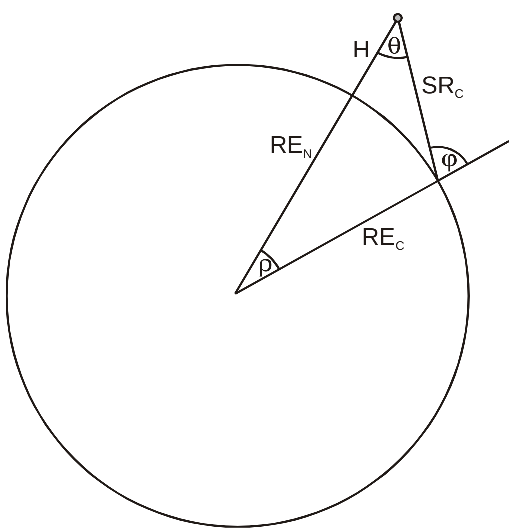

Figure i.1: *Range from Earth center to spacecraft*

ρ	Central Earth angle
τ	Range gate delay
φ	Incidence angle
ω_E	Earth rotation rate (degrees)
$\{vx_0,vy_0,vz_0\}$	Spacecraft Platform Velocity vector for start of image frame
$\{vx_1,vy_1,vz_1\}$	Spacecraft Platform Velocity vector for center of image frame
$\{vx_2,vy_2,vz_2\}$	Spacecraft Platform Velocity vector for end of image frame
$\{x_0,y_0,z_0\}$	Spacecraft Platform Position vector for start of image frame
$\{x_1,y_1,z_1\}$	Spacecraft Platform Position vector for center of image frame
$\{x_2,y_2,z_2\}$	Spacecraft Platform Position vector for end of image frame

BM	Beam mode
BN	Beam Number
C	Speed of Light (m/s)
d	Straight line distance between first and last state vector
D_A	Swath width in azimuth direction
D_R	Swath width in range direction
DTR	Degrees to radians conversion factor
ECC_E	Eccentricity of the Earth
ECC2	Eccentricity of the Earth squared
f_{D0}	Doppler centroid frequency near range
f_{D1}	Doppler centroid frequency mid-range
f_{D2}	Doppler centroid frequency far range
f_S	Complex Sampling Rate (MHz)
GHA_F	Greenwich hour angle for start of imaging period
GHA_L	Greenwich hour angle for end of imaging period
GHA_M	Greenwich hour angle for middle of imaging period
H	Spacecraft Altitude; Height of platform above ellipsoid surface
L	Number of lines in the image product
L_C	Center line number in the image
ORBIT	Orbit number
P	Number of pixels in the image product
P_C	Center pixel number in the image
PF	Platform Name { E1, E2, R1, J1 }
PIXSIZ	Pixel spacing
PM	Processor Mode
PT	Projection Type
r	Radius at first spacecraft position
RE	Equatorial Earth radius
RE_C	Radius of the Earth at the center of the image
RE_N	Radius of the Earth at the nadir (center of the image time)
RP	Polar Earth radius
R_{SC}	Range from Earth center to spacecraft ; equal to mag(scVec[1])
RTD	Radians to degrees conversion factor
SR_C	Slant Range to the center of the image
SR_F	Slant Range to the first pixel of the image
SR_L	Slant Range to the last pixel of the image
V_R	Relative velocity between Radar and Target
V_{SW}	Velocity of the swath on the ground

LIST OF ACRONYMS AND ABBREVIATIONS

ASF	– Alaska SAR Facility
ATDR	– Attitude Data Record
BM	– Beam Mode
BN	– Beam Number
CCSD	– Computer compatible signal data
CEOS	– Committee on Earth Observation Systems
DQSR	– Data Quality Summary Record
DS	– Data Spacing
DSSR	– Dataset Summary Record
DTR	– Degrees to Radians conversion factor
DWP	– Data window position
EBF	– Earth Body Fixed
EH	– Extended high incidence beam
EL	– Extended low incidence beam
FACDR	– Facility Related Data Record
FDR	– Leader File Descriptor Record
FN	– Fine Beam
GCD	– Geocoded
GEI	– Geocentric Equatorial Inertial
GHA	– Greenwich Hour Angle (degrees)
GRF	– Georeferenced (ground range or slant range)
MPDR	– Map Projection Data Record
PDHR	– Processed Data Histogram Record
PF	– Platform Name
PIXSIZ	– Pixel Size (spacing) (m)
PM	– Processor Mode
PPDR	– Platform Position Data Record
PRF	– Pulse Repetition Frequency (Hz)
PT	– Projection Type
RADDR	– Radiometric Data Record
RSR	– Range Spectra Record
RTD	– Radians to Degrees conversion factor
SDHR	– Signal Data Histogram Record
SN	– ScanSAR narrow beam
ST	– Standard Beam
SW	– ScanSAR wide beam
WD	– Wide Beam

1. INTRODUCTION

This manual is the result of developing software tools for radar images and looking into their data quality for several years. Often, troubleshooting problems related to theses activities requires the calculation and derivation of CEOS metadata values. The set of formulas has grown with time into a larger document. With some organizational work it made it to the manual in its present form.

This manual is organized in the following way:
In chapter *ASF CEOS products* the CEOS records in which the metadata values are stored are described in detail.
The *constants and fixed parameters* used in the processing are defined and the ASF naming conventions are explained.
The *CEOS structures* have various fields that are constant. Furthermore, the structures contain fields with information on image name, processing as well as product type and size.
The *geometric calculations* are the core of this manual as they describe the way how to determine the most important metadata values.
The appendices expand on details of the geometric calculations: the *mathematical background* (basics of vector mathematics, spherical coordinates and the definition of an ellipse), the *transformations for slant range and ground range*, the *Earth latitudes and coordinate systems*, the *geolocation algorithm* and the *ASF image orientation*.
In the electronic version , green colored terms indicate cross links within the document

2. ASF CEOS PRODUCTS

The Alaska SAR Facility (ASF) uses the Committee on Earth Observation Systems (CEOS) standards for image product formats. This standard defines two file types, a self describing data file (.D) and an external leader file (.L) containing relevant information on the product's origin and processing history.

The CEOS structures included in the data files are collectively referred to as the CEOS "wrapper", shown in Figure 2.1. The image file descriptor record, which provides information on the size and layout of the data in the remaining lines of the file, precedes the data file. Each line of the data file starts with a record header which gives information about the record it precedes including the record count and record length.

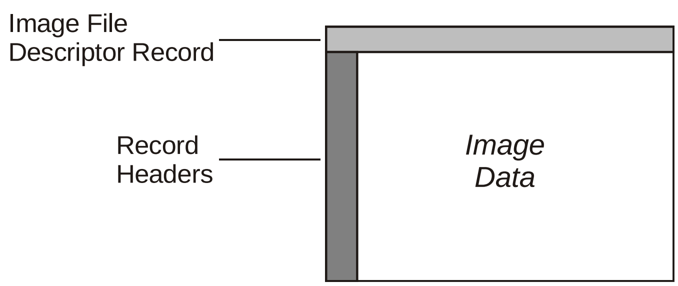

Figure 2.1**:** *CEOS wrapper*

The CEOS leader file contains a varying number of metadata records depending on the type of product:

CCSD – Computer Compatible Signal Data,
GRF – Georeferenced (ground range or slant range), or
GCD – Geocoded.

The occurrence of each of the record types is as follows:

Record type	**Acronym**	**CCSD**	**GRF**	**GCD**
Leader File Descriptor	FDR	x	x	x
Dataset Summary	DSSR	x	x	x
Map Projection Data	MPDR			x
Platform Position Data	PPDR	x	x	x
Attitude Data	ATDR	x	x	x
Radiometric Data	RADDR		x	x
Data Quality Summary	DQSR		x	x

Record type	Acronym	CCSD	GRF	GCD
Signal Data Histogram	SDHR	x	x	x
Processed Data Histogram	PDHR		x	x
Range Spectra	RSR	x	x	x
Facility Related Data	FACDR	x	x	x

Table 2.1: *CEOS leader file record*

This document will be restricted to the derivation of the most commonly used fields of these record structures. For a complete listing of the CEOS fields used by ASF please refer to the ASF Product Format Specification document (Bicknell, 1997).

3. CONSTANTS AND FIXED PARAMETERS

In this chapter, the constants and fixed parameters used during the SAR processing are discussed in detail.

A small number of universal constants need to be defined. The radar satellite data received and processed at ASF are acquired with different sensors operating in various modes. These data sets have sensor/mode specific constants. Furthermore, there are fixed parameters that describe the algorithm used for the SAR processing. Finally, ASF stores its products with unique names that contain information about the data source and the type of processing.

3.1 UNIVERSAL CONSTANTS

Before entering the core of calculating some constants need to be defined for the use in various calculations. These are given in Table 3.1.

Identifier	Description	Value
C	Speed of Light (m/s)	2.99792458 E8
RE	GEM6 Model Equatorial Earth Radius (km)	6378.144
RP	GEM6 Model Polar Earth Radius (km)	6356.7549
ECC_E	GEM6 Model Eccentricity of the Earth	8.1827385 E-2
ECC2	Eccentricity of the Earth squared	ECC_E * ECC_E
ω_E	Earth Sidereal Rotation Rate (degrees/sec)	0.00417807442
RTD	Radians to Degrees Conversion Factor	180.0 / π
DTR	Degrees to Radians Conversion Factor	π / 180.0

Table 3.1*:* *Definitions of universal constants*

3.2 SENSOR/MODE BASED CONSTANTS

A certain number of the fields in the CEOS structures is determined directly from the sensor and mode that it is operated in. ASF receives and processes data from four different SAR platforms.

Platform	Acronym	Flight Agency
European Remote Sensing Satellite – 1	ERS-1	European Space Agency (ESA)
European Remote Sensing Satellite – 2	ERS-2	European Space Agency (ESA)
Japanese Earth Resources Satellite - 1	JERS-1	National Space Development Agency (NASDA)
Radar Satellite - 1	RADARSAT-1	Canadian Space Agency (CSA)

Table 3.2: *Satellite acronyms*

The ERS-1, ERS-2, and JERS-1 satellites only operate in a single fixed mode. The Radarsat-1 satellite can be operated in 26 different modes, each of which provides different image geometry characteristics. The modes are classified as standard (ST), fine (FN), wide (WD), extended high incidence (EH), extended low incidence (EL), ScanSAR wide (SW), and ScanSAR narrow (SN). Each of these modes has from 1 to 7 different beams that can be used, as indicated in the Radarsat imaging modes Table 3.3**.**

Beam Name	Range Sampling Rate	PRF	Range Bandwidth	Electronic Boresight	N Pulses
ST1	18.46	1316	17.48	20.366	7
ST2	18.46	1276	17.48	24.2	7
ST3	12.92	1354	11.78		8
ST4	12.92	1315	11.78	31.875	8
ST5	12.92	1275	11.78	34.064	8
ST6	12.92	1334	11.78	38.20718	9
ST7	12.92	1276	11.78	40.438	9
WD1	12.927	1270	11.58		7
WD2	12.927	1310	11.58		8
WD3	12.927	1350	11.58		9
FN1	32.317	1300	30.002		8
FN2	32.317	1270	30.002		8
FN3	32.317	1330	30.002		9
FN4	32.317	1315	30.002		9
FN5	32.317	1300	30.002		9
EL1	12.927	1345	11.58		?

Beam Name	Range Sampling Rate	PRF	Range Bandwidth	Electronic Boresight	N Pulses
EH1	12.927	1345	11.58		10
EH2	12.927	1307	11.58		10
EH3	12.927	1270	11.58		10
EH4	12.927	1245	11.58		11
EH5	12.927	1307	11.58		11
EH6	12.927	1270	11.58		11
SWA	12.927	VAR	11.58		*
SWB	12.927	VAR	11.58		*
SNA	12.927	VAR	11.58		*
SNB	12.927	VAR	11.58		*

Table 3.3: *Radarsat imaging modes*

Using the table above as a reference, we now define two new variables, the beam mode *BM* and beam number *BN*.

BM takes the value of the first two characters of the beam name, while *BN* takes the value of the last. For ERS-1, ERS-2, and JERS-1, *BM* = ST and *BN* = 1.

What follows is a table of the most important SAR sensor parameters for each of the satellite data that ASF processes. This table gives the parameter name, the CEOS record the value is stored in, the units, and the values for each of the platforms. Also, the identifier used to reference the value later in this document is given in the last column.

Parameter	CEOS record	Units	ERS1/2	RSAT-1	JERS-1	Identifier
Antenna Length		m	10	15	11.9	
Antenna Width		m	1	1.5	2.4	
Frequency of Carrier Wave	DSSR	GHz	5.3	5.3	1.275	
Wavelength	DSSR	m	0.0565646	0.0565646	0.23513	λ
Range Pulse Length	DSSR	μsec	37.1	42	35	
Quantization Bits	DSSR, FACDR	bits	5	4	3	
Chirp Slope	DSSR	Hz/sec	4.19 E11			

Parameter	CEOS record	Units	ERS1/2	RSAT-1	JERS-1	Identifier
Range Sampling Rate	DSSR	MHz	18.96	VAR*	17.08	f_s
Electronic Boresight	DSSR	deg	20.3	VAR*		
Pulse Repetition Frequency[1]	DSSR, FACDR	Hz	1679.9	VAR*	1555	PRF
Elevation Beamwidth	DSSR	deg	5.4			
Azimuth Beamwidth	DSSR	deg	0.2852			
Azimuth Bandwidth	DSSR	Hz	1260	960	1157	
Range Bandwidth	DSSR	MHz	15.55	VAR*	15	
Data Rate	FACDR	Mbits/sec	105			

* These parameters vary based on the imaging mode, see previous table
[1] PRF varies based on spacecraft altitude; the values given are averages

Table 3.4: *SAR sensor fixed parameters*

3.3 Parameters Fixed by Processing System

Several fields of the dataset summary record contain information regarding the SAR correlation algorithm that was used on the data. Depending on the type of SAR signal being processed and the type of output image desired, one of three different SAR correlation algorithms are used by ASF. Also, two different weighting functions are used.

Field Name	Field Value	Conditional
facility id	ASF-PGS	
system id	{ASP,SSP-2,SSP-1,PREC}	
version id	<contains system version>	
algorithm id	DERAMP FFT RANGE DOPPLER CHIRP SCALING	BM = SW, SN PM = R, S Otherwise
azimuth weighting	COSINE SQUARED ON PEDESTAL KAISER	ASP system other systems
range weighting	COSINE SQUARED ON PEDESTAL KAISER	ASP system other systems

Table 3.5: *SAR correlation algorithms and weighting functions*

3.4 About ASF Image Names

The ASF product names are designed to uniquely identify the source of the SAR data and the amount and type of processing that has been performed on the data.

Filename Syntax: ***PPRRRRRFFFMDSvvv.T***

PP	Platform name (E1, E2, R1, J1) indicates platform that collected the data
RRRRR	Satellite revolution (or orbit) number for this data take
FFF	Fixed frame numbering scheme relative to ascending node (001-900)
M	Projection type (S – slant range, G – ground range, or geocoded: U – Universal Transverse Mercator; P – Polar Stereographic; L – Lambert)
D	Data pixel spacing = 2**D * 6.25 meters (0 also means natural spacing)
S	Processor mode (S – Standard, C – CCSD, X – Complex, R – RAMP)
vvv	Version of product type
T	File type (L – CEOS Leader, D – CEOS Data)

Each of these fields in an image name will play a role in determining the correct values to be placed in certain CEOS fields. The following table defines the identifiers that will be used to reference the values in the remainder of this document.

Identifier	Description	Value	
PF	Platform Name	E1, E2, J1, R1	
ORBIT	Orbit Number	5-digit Number	
FRAME	Frame Number	1 - 900	
PT	Projection Type	S, G, U, P, L	
DS	Data Spacing Number	0,1,2, …	
PM	Processor Mode	C, X, S, R	
PIXSIZ	Pixel Spacing	Natural[1] 25.0 $6.25*2^{DS}$	if *PM* = C, X if *PM* = R otherwise

[1] See section 5.5 for discussion of natural pixel spacings

Table 3.6*:* *Global parameters determined from the image name*

4. CEOS STRUCTURES

In this chapter the ASF CEOS constants, image name fields, processing flags and product type/size fields that are filled into the various CEOS metadata records are introduced.

4.1 ASF CEOS CONSTANTS

Many of the fields in the CEOS records are always set to the same value. These are the "constant" fields. What follows is a list of the field names and values by record type. All of these values were taken from the ASF Product Format Specifications (Bicknell, 1997).

CEOS record	Field Name	Field Value
DSSR	ellipsoid designator	GEM06
	ellipsoid semi-major	6378.144 km
	ellipsoid semi-minor	6356.7459 km
	Earth mass constant	398600.5 (unit???)
	gravitational constant	9.8 m/s^2
	ellipsoid J2 parameter	0.0010826
	ellipsoid J3 parameter	-0.0000025
	ellipsoid J4 parameter	-1610000.0
	terrain height	0.0
	number of SAR channels	1
	motion compensation indicator	00
	range pulse code specifier	LINEAR FM CHIRPS
	quantizer description	UNIFORM I,Q
	baseband flag	YES
	range compressed flag	NOT
	receiver gain like polarization	0.0
	receiver gain cross polarization	0.0
	mechanical boresight	0.0
	radiometric stretch term 1	0.0
	radiometric stretch term 2	0.0
	range compression designator	SYNTHETIC CHIRP
	clutterlock flag	YES

CEOS record	Field Name	Field Value
PPDR	orbital elements designator	KEPLERIAN ORBITAL ELEMENTS
	number of data sets	3
	reference coordinate system	GEOCENTRIC EQUATORIAL INERTIAL
ATDR	pitch rate flag	blank
	roll rate flag	blank
	yaw rate flag	blank
FACDR	look energy normalized flag	NOT
	image processing type	SF
	terrain height above geoid	0.0
	estimated noise floor	-21.0
	radiometric resolution	0.1
	clutterlock flag	YES
	on-board range compression flag	NOT

Table 4.1: *Constants fields in the CEOS records*

4.2 IMAGE NAME FIELDS

Several different image "name" fields occur within the CEOS records. The contents of these fields can be determined from the product name as follows:

Let "fullname" be the image name without the version number,
"codename" be the fullname without the platform string and orbit number.

CEOS record	Field Name	Field Value(s)	Conditional
DSSR	product id	<fullname>	
	facility code	<codename>	
	orbital revolution	ORBIT	
	mission id	{ERS-1,ERS-2,JERS-1,RADARSAT-1}	
	sensor id	ERS-1-C - -VV ERS-2-C - -VV JERS-1-L - -VV RADARSAT-1-C - -HH	if *PF* = E1 if *PF* = E2 if *PF* = J1 if *PF* = R1

CEOS record	Field Name	Field Value(s)	Conditional
FACDR	data take id	<fullname>	
	image id	<codename>	
	site name	<codename>	
	platform	{ERS-1,ERS-2,JERS-1,RADARSAT-1}	
	sensor and mode	ERS-1-C - -VV ERS-2-C - -VV JERS-1-L - -VV RADARSAT-1-C - -HH	if *PF* = E1 if *PF* = E2 if *PF* = J1 if *PF* = R1

Table 4.2: *Name fields in the CEOS records*

4.3 PROCESSING FLAGS

Throughout the CEOS metadata, several processing/informational flags occur. These are set based on the type and amount of processing that has been performed (among other things) as indicated by the following Table 4.3.

CEOS record	Field Name	Field Value(s)	Conditional
DSSR	ascending/descending flag	A D	if (frame < 226 or frame > 674) otherwise
	time direction (range)	blank INCREASE	if *PT* = U, P, L otherwise
	time direction (azimuth)	blank INCREASE DECREASE	if *PT* = U, P, L if *PM* = C, R otherwise
	line content indicator	OTHER RANGE	if *PT* = U, P, L otherwise
	sensor clock angle	90.0 -90.0	for right looking for left looking
	echo tracker flag	blank {ON,OFF}	if *PM* = C otherwise

CEOS record	Field Name	Field Value(s)	Conditional
FACDR	ascending/descending flag	ASCENDING DESCENDING	if (frame < 226 or frame > 674) otherwise
	spacecraft roll flag	{0,-1}	
	spacecraft yaw flag	{0,-1}	
	spacecraft pitch flag	{0,-1}	
	spacecraft roll rate flag	{0,-1}	
	spacecraft yaw rate flag	{0,-1}	
	spacecraft pitch rate flag	{0,-1}	
	ephemeris type identifier	{R,P,D}	
	ground/slant range indicator	SLANT GROUND	if *PT* = S otherwise
	deskew flag	NOT YES	if *PT* = S otherwise

Table 4.3: *Processing and information flags in CEOS metadata*

4.4 PRODUCT TYPE/SIZE FIELDS

The dataset summary record contains a product type field, that takes the values:

SCANSAR	if *BM* = SW, SN
RAMP	if *PM* = R
CCSD	if *PM* = C
COMPLEX	if *PM* = X
{FULL,CEOS_LOW}	otherwise

The following table captures the relationship between product type, pixel spacings, and the number of lines and samples in detected image products. The number of lines and samples (total lines / total samples from the FACDR) and pixel spacing (line spacing / pixel spacing from the DSSR and pixel spacing in azimuth / pixel spacing in range from the FACDR) are decided by beam and product type as

Beam Mode	Product Type	PIXSIZ	NL, NS
ST	FULL CEOS_LOW	12.5 100.0	8192, 8192 1024, 1024
WD	FULL CEOS_LOW	25.0 200.0	6144,6144 768, 768
FN	FULL CEOS_LOW	6.25 50	8192, 8192 1024, 1024
EL	FULL CEOS_LOW	12.5 100	12288, 12288 1536, 1536
EH	FULL CEOS_LOW	12.5 100.0	8192, 8192 1024, 1024
SW	SCANSAR SCANSAR SCANSAR	50 100 400	11000, 11000 5500, 5500 1375, 1375
SN	SCANSAR SCANSAR SCANSAR	50 100 400	6144, 6144 3072, 3072 768, 768

Table *4.4:* *Pixel spacing for different Radarsat modes*

Note: This table does not apply to RAMP, CCSD, or COMPLEX
For a description of the sizes for those images pleases refer to Bicknell (1997).

The dataset summary record also contains fields for the center line and center pixel of an image. These come directly from the formulas:

$$L_C = \frac{\text{total lines}}{2.0} + 0.5$$
$$P_C = \frac{\text{total pixels}}{2.0} + 0.5 \qquad (4.1)$$

5. GEOMETRIC CALCULATIONS

5.1 CEOS FIELDS WHICH ARE ASSUMED TO BE CORRECT

In order to derive the scene dependent parameters for an image, we need a basic set that is assumed to be correct. What follows is a discussion of these parameters:

Slant Ranges

The first set of critical parameters are the slant ranges from the satellite to the Earth's surface. These are the straight line distances from the satellite to the Earth's surface at the first pixel, center pixel, and last pixel of the swath. From the slant ranges, we calculate MANY of the other geometric parameters.

Slant Range to the First Pixel in the image swath (SR_F):
Taken directly from the FACDR field of the same name.

Slant Range to the Last pixel of the image swath (SR_L):
Taken directly from the FACDR field of the same name.

Slant Range to the Center of the image swath (SR_C):
In order to calculate various parameters for the center of an image, we must know the slant range distance to the center of the image swath. For the case where the image is still in the slant range projection, (PT = S or PM = R), it is a simple calculation of :

$$SR_C = \frac{(SR_L - SR_F)}{2} + SR_F \tag{5.1}$$

For the case where the image is in ground range projection, it is a bit more challenging. However, using two applications of the law of cosines, the ground range to a pixel of interest can be derived, and from that the slant range to the same pixel can be determined (see Appendix B). This procedure requires the knowledge of the following parameters:

R_{SC} = Range of spacecraft from center of Earth
SR_F = Slant Range to the first image pixel
RE_C = Earth radius at the center of the image
GR_X = Ground range from start of swath to point of interest

The value for SR_F is already known. R_{SC} is simply the magnitude of the center spacecraft position state vector (see next section). And, the ground range from the start of the swath to the center pixel is

$$GR_X = \frac{P_C \cdot PIXSIZ}{1000.0} \tag{5.2}$$

The problem is the value of RE_C. There are two different approaches to take. The first approach, adopted by the check_ceos program, is to add the FACDR *Earth radius at the center of the image* field to the list of CEOS fields that are assumed to be correct. The second approach, would be to calculate and use the value of RE_N as an approximation to the Earth radius at the center of the image.

Aside:
In practice, the value for the slant range to the first pixel is taken directly from the FACDR. However, its correctness can be verified. In order to get the slant range, we need the round-trip time τ is needed, for the beam to go from the sensor to the ground and back. This quantity, referred to as the range gate delay, is measured in microseconds. Since the signal moves at the speed of light, the slant range to the first pixel, in km, is given by:

$$SR_F = \frac{\tau \cdot C}{2.0 \cdot 10^6 \cdot 1000.0} \tag{5.3}$$

where C is the speed of light.

In order to understand the calculation of τ, we need to define a few quantities:

N Pulses (n):
Because of the speed at which the sensor is transmitting signals, several pulses are sent out by the satellite before the first echo is listened for. The number of pulses in the air, n, varies by platform and mode. For ERS-1 and ERS-2 n = 9, for JERS-1 n = 7, and n for various Radarsat modes is given in the RADARSAT Imaging Modes table in Table 3.3.

Data Window Position (DWP):
In addition to the number of pulses in the air, the data window position must be taken into account when calculating the range gate delay. The DWP is the value in microseconds from the near-set PRF trigger to the start of the digitization of the return echo. In other words, it is the amount of time after a pulse is sent out that passes by before recording an echo from the ground. The CEOS fields that contain the DWP are:

FACDR	datawin
DSSR	rng_gate

The total time delay τ for a transmitted pulse to return an echo to the spacecraft including both the DWP and the number of pulses (echoes) in the air is calculated by

$$\tau = DWP + \frac{n}{PRF} \cdot 10^6 + \Delta \tag{5.4}$$

Note: Δ is an arbitrary correction needed to compensate for the (unknown) delay in the radar electronics. This fixed quantity is added to τ so that the SR_F equation above will hold.

The CEOS Fields that contain the range gate delay τ, are:

FACDR	rangegd
DSSR	rng_gate1

Spacecraft State Vectors

The location of the satellite must be known in order to calculate anything about what it is imaging on the ground. Thus, position and velocity vectors for the spacecraft are given in the CEOS PPDR structure. The three sets of vectors given correspond to the start, center, and end of the imaging period. Values are in geocentric equatorial inertial coordinates with units of km for positions and m/s for velocities.

One needs to be aware of the image time in the azimuth direction (see section 4.3) when determining the proper state vector to use. If the time direction is increasing, the first state vector in the PPDR corresponds to the “top” of the image product, while the last vector refers to the “bottom” of the image product. However, if the time direction is decreasing, then the opposite is true, i.e. the first state vector is for the “bottom” of the image and the last state vector is for the “top” of the image.

Identifiers used in this document to denote these quantities are as follows:

$\{x_0, y_0, z_0\}$	Spacecraft Platform Position vector for start of image frame
$\{x_1, y_1, z_1\}$	Spacecraft Platform Position vector for center of image frame
$\{x_2, y_2, z_2\}$	Spacecraft Platform Position vector for end of image frame
$\{vx_0, vy_0, vz_0\}$	Spacecraft Platform Velocity vector for start of image frame
$\{vx_1, vy_1, vz_1\}$	Spacecraft Platform Velocity vector for center of image frame
$\{vx_2, vy_2, vz_2\}$	Spacecraft Platform Velocity vector for end of image frame

Image Timings

In order to determine when the position and velocity vectors occur, the time fields from the PPDR must also be used. Assuming that the state vectors in the PPDR are correct means assuming that the timing given is correct as well. The time values *year*, *month*, *day of the month*, *gmt_day*, and *gmt_sec*, refer to the first spacecraft vector. An additional field, the *data interval*, gives the time interval between state vectors. So, the first state vector corresponds to the time given, the second is data interval seconds later than the first and the last is two times *data interval* seconds later than the first.

Greenwich Hour Angle

The Greenwich hour angle (GHA) provides the means to convert from geocentric Earth-centered inertial coordinates to Earth body fixed coordinates (see Appendix C for details). Since the GHA measures the Earth's rotation from a fixed position, it changes along the image swath.

Two fields in the PPDR provide the means to determine the GHA during an imaging period. They are the *hour angle* and the *data interval*. The first is the base GHA for the beginning of the imaging period. The second is the time difference between the start, center, and ending state vectors. Using these we calculate the GHAs as:

$$\begin{aligned} \mathrm{GHA}_F &= \text{hour angle} \\ \mathrm{GHA}_M &= \text{hour angle} + \omega_E \cdot \text{data interval} \\ \mathrm{GHA}_L &= \text{hour angle} + \omega_E \cdot 2 \cdot \text{data interval} \end{aligned} \tag{5.5}$$

where ω_E is the Earth rotation rate.

The same comments about the image time directions that applied to the state vectors apply to the GHA.

Doppler centroid frequencies

In order to accurately determine geolocations for images, the Doppler centroid frequency must be known. Coefficients for a quadratic function to calculate the Doppler parameters for an image are given in both the DSSR and the FACDR. The fields pertinent here are:

DSSR: Cross Track Doppler Frequency 1, 2 and 3
FACDR: Doppler Frequency near, slope and quadratic

$$
\begin{aligned}
f_{D0} &= \text{Doppler frequency near} \\
f_{D1} &= f_{D0} + P_C \cdot \text{slope} + P_C{}^2 \cdot \text{quadratic} \\
f_{D2} &= f_{D0} + P \cdot \text{slope} + P^2 \cdot \text{quadratic}
\end{aligned} \tag{5.6}
$$

where P_C is the center pixel number and P is the number of pixels in the image.

5.2 NADIR CALCULATIONS

Radius of the Earth and longitude at the nadir

The nadir is the point on the surface of the Earth directly beneath the platform at the center of the imaging time. A *geocentric* nadir is determined by the straight line from the platform to the center of the Earth. A *geodetic* nadir is determined by the straight line from the platform to the perpendicular intersection with the Earth's surface.

For the geocentric nadir point, geolocation is determined by the intersection of the spacecraft vector with the surface of the Earth. Let $\vec{R}_{SC} = \{x_1, y_1, z_1\}$, denote the spacecraft position vector for the center of the image after rotation into Earth body fixed (EBF) coordinates. Then, the spacecraft radius from the center of the Earth is:

$$R_{SC} = \mathrm{mag}(\vec{R}_{SC}) \tag{5.7}$$

Using cartesian to spherical conversions, the geocentric and then geodetic latitude and the longitude of the geocentric nadir point can be determined by

$$\begin{aligned}
\theta_C &= \arcsin \frac{z_1}{R_{SC}} && \text{(Geocentric latitude at nadir)} \\
\theta_D &= \arctan \frac{\tan \theta_C}{1 - ECC_E^2} && \text{(Geodetic latitude at nadir)} \\
\mathrm{lon} &= \arctan \frac{y_1}{x_1} && \text{(Geodetic and geocentric longitude at nadir)}
\end{aligned} \tag{5.8}$$

where ECC_E is the Earth eccentricity.

Since the difference from geocentric to geodetic nadir points is independent of longitude, this is the final form for the spacecraft's longitude at a geocentric as well as a geodetic nadir. However, further derivation is required to reach the final form of the spacecraft's geodetic latitude at a geodetic nadir.

Knowing the geocentric latitude of the nadir, the Earth radius at the geocentric nadir is calculated by

$$RE_N = \frac{(RE \cdot RP)}{\sqrt{(RP \cdot \cos\theta_C)^2 + (RE \cdot \sin\theta_C)^2}} \tag{5.9}$$

where RE is the equatorial Earth radius and RP is the polar Earth radius.

Spacecraft altitude and latitude at the nadir

Based on the two types of nadir points, there are also two types of spacecraft altitudes. Given the radius to the spacecraft R_{SC}, and the radius of the Earth at the geocentric nadir RE_N, the *geocentric* altitude of the spacecraft above the ellipsoid is a simple difference of the two:

$$H' = R_{SC} - RE_N \tag{5.10}$$

Thus, H' is measured along the shortest distance to the Earth's center; but not the shortest distance to the Earth's surface. Figure 5.1 shows that H, the *geodetic* spacecraft altitude is not the same as H'.

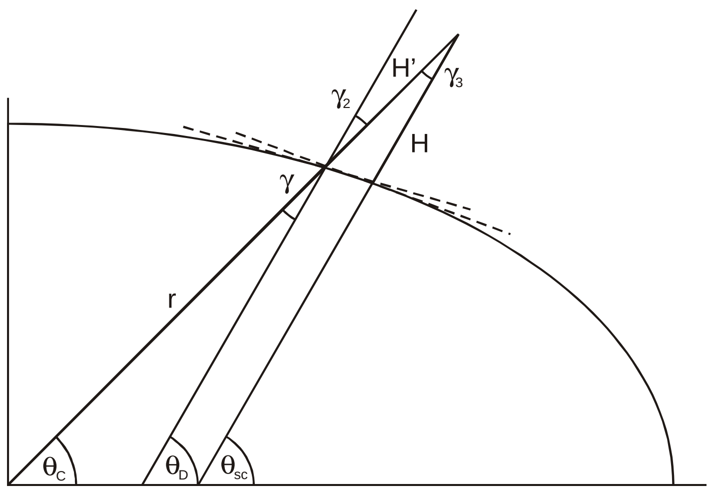

Figure 5.1: *Geodetic spacecraft altitude*

H can be derived by realizing that the angle γ is equal to the difference from the geodetic to geocentric latitudes calculated above, i.e. $\gamma = \theta_D - \theta_C$. The approximation that $\gamma_3 \approx \gamma_2 = \gamma$ leads to $H' = R_{SC} - RE_N$ and finally to

$$\cos\gamma = \frac{H}{H'} \quad \Rightarrow \quad H = (R_{SC} - RE_N) \cdot \cos\gamma \qquad \text{(geodetic spacecraft altitude)} \tag{5.11}$$

This then, is the spacecraft altitude at the center of the image as calculated in the ASF CEOS metadata. Just as H' does not measure the shortest distance to the Earth's surface, θ_D does not measure the true geodetic latitude of the spacecraft's position. The true value is depicted as θ_{SC}.

Consider the distance r, which completes the triangle with H' and H. This is the distance between the intersection of the vector defined by θ_D and the vector defined by θ_{SC}. It can be approximated by

$$r = (R_{SC} - RE_N) \cdot \sin\gamma_3 \approx H \cdot \sin\gamma \tag{5.12}$$

Since r is the distance along the (approximately flat) surface of the Earth, r / RE_N gives an estimate of the difference in the angles from the previously calculated latitude to the latitude that we are searching for. That is,

$$\theta_D - \theta_{SC} \approx \frac{r}{RE_N} \tag{5.13}$$

Finally then, θ_{SC} can be calculated using

$$\theta_{SC} = \theta_D - \frac{r}{RE_N} \quad \Rightarrow \quad \theta_{SC} = \theta_D - \frac{\sin\gamma \cdot H}{RE_N} \quad \text{(geodetic nadir latitude)} \tag{5.14}$$

This then, is the final form of the equation for computing the spacecraft latitude at the nadir.

5.3 Georeferencing Information

Geolocations for the image corners are given in the FACDR. Also, a geolocation for the center of the image is included in both the FACDR and the DSSR. Given the slant range to a pixel, the Doppler frequency centroid, and the spacecraft state vector, one can solve for the geolocation of a pixel using an iterative method of adjusting satellite look and squint angles. This method is given in full detail in Appendix D.

It should be mentioned that if the image has already been Doppler deskewed (see processing flags in section 4.3), then the geolocation routine should be called using Doppler centroids of 0 rather than the values calculated in subsection Doppler frequency centroids.

5.4 Image Center Calculations

Radius of the Earth at the Center of the image

Let lat be the geocentric latitude at the center of the image, RE the equatorial Earth radius, and RP the polar Earth radius, then

$$RE_C = \frac{RE \cdot RP}{\sqrt{(RP \cdot \cos\ lat)^2 + (RE \cdot \sin\ lat)^2}} \tag{5.15}$$

Central Look Angle

The look angle at the center of an image θ, is defined as the elevation angle from the nadir point to the center of the image subtended by the satellite position. Given the range from Earth center to spacecraft R_{SC} and the slant range to the image center SR_C, it can be found by the law of cosines as follows.

$$\begin{aligned} RE_C^{\ 2} &= R_{SC}^{\ 2} + SR_C^{\ 2} - 2 \cdot R_{SC} \cdot SR_C \cdot \cos\theta \\ \cos\theta &= \frac{R_{SC}^{\ 2} + SR_C^{\ 2} - RE_C^{\ 2}}{2 \cdot R_{SC} \cdot SR_C} \end{aligned} \tag{5.16}$$

using the approximation $R_{SC} = RE_C + H$ with the geodetic spacecraft altitude H leads to

$$\begin{aligned} \cos\theta &= \frac{(RE_C + H)^2 + SR_C^{\ 2} - RE_C^{\ 2}}{2 \cdot SR_C \cdot (RE_C + H)} \\ &= \frac{RE_C^{\ 2} + 2 \cdot H \cdot RE_C + H^2 + SR_C^{\ 2} - RE_C^{\ 2}}{2 \cdot SR_C \cdot (RE_C + H)} \\ &= \frac{SR_C^{\ 2} + 2 \cdot H \cdot RE_C + H^2}{2 \cdot SR_C \cdot (RE_C + H)} \quad \text{[equation D4 from Olmsted (1993)]} \\ &= \frac{2 \cdot H \cdot RE_C + 2 \cdot H^2 - H^2 + SR_C^{\ 2}}{2 \cdot SR_C \cdot (RE_C + H)} \\ &= \frac{2 \cdot H \cdot (RE_C + H) + SR_C^{\ 2} - H^2}{2 \cdot SR_C \cdot (RE_C + H)} \\ &= \frac{2 \cdot H \cdot (RE_C + H) + SR_C^{\ 2} - H^2}{2 \cdot SR_C \cdot R_{SC}} \quad \text{[approximating } R_{SC} = RE_C + H \text{]} \end{aligned} \tag{5.17}$$

now let, $\Delta = \dfrac{SR_C^{\ 2} - H^2}{2 \cdot R_{SC}}$

then, $H + \Delta = \dfrac{2 \cdot H \cdot R_{SC} + SR_C^{\ 2} - H^2}{2 \cdot R_{SC}}$

and, $\dfrac{H + \Delta}{SR_C} = \dfrac{2 \cdot H \cdot R_{SC} + SR_C^{\ 2} - H^2}{2 \cdot R_{SC} \cdot SR_C}$

so, finally,

$$\theta = \arccos\left(\frac{H+\Delta}{SR_C}\right) \tag{5.18}$$

Central incidence angle

The central incidence angle φ is the angle between the incident beam, and the extension of a vector from the center of the Earth through the center of the imaging swath. First, the relationship between the central incidence angle φ, the look angle θ, and the central Earth angle ρ needs to be established. Now, $A = 180 - (\theta + \rho)$. Also, $\varphi + A = 180$. This leads to $\varphi + 180 - (\theta + \rho) = 180$, or $\varphi = \theta + \rho$ [equation D2 from Olmsted (1993)]. Given the range from Earth center to spacecraft R_{SC}, the slant range to image center SR_C, the Earth radius at image center RE_C, the geodetic spacecraft altitude H and using the equation developed in Appendix B for ρ, it can be seen that

$$\begin{aligned}
\cos\rho &= \frac{1 + \dfrac{R_{SC}{}^2}{RE_C{}^2} - \dfrac{SR_C{}^2}{RE_C{}^2}}{\dfrac{2 \cdot R_{SC}}{RE_C}} = \frac{RE_C{}^2 + R_{SC}{}^2 - SR_C{}^2}{2 \cdot R_{SC} \cdot RE_C} \\
&= \frac{RE_C{}^2 + (RE_C + H)^2 - SR_C{}^2}{2 \cdot R_{SC} \cdot RE_C} = \frac{2 \cdot RE_C{}^2 + 2 \cdot RE_C \cdot H + H^2 - SR_C{}^2}{2 \cdot R_{SC} \cdot RE_C} \\
&= \frac{2 \cdot RE_C \cdot (RE_C + H) + H^2 - SR_C{}^2}{2 \cdot R_{SC} \cdot RE_C} = \frac{2 \cdot RE_C \cdot R_{SC} + H^2 - SR_C{}^2}{2 \cdot R_{SC} \cdot RE_C} \\
&= 1 + \frac{H^2 - SR_C{}^2}{2 \cdot R_{SC} \cdot RE_C} = 1 - \frac{SR_C{}^2 - H^2}{2 \cdot R_{SC} \cdot RE_C} \\
&= 1 - \frac{\Delta}{RE_C} \\
\rho &= \arccos\left(1 - \frac{\Delta}{RE_C}\right)
\end{aligned} \tag{5.19}$$

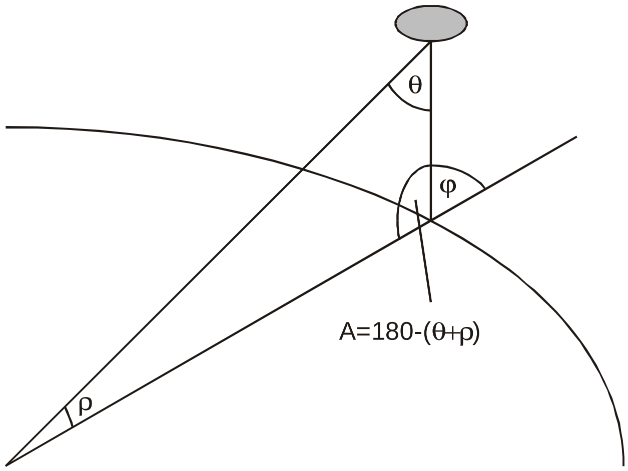

Figure 5.2: *Central incidence angle*

Finally, the incidence angle is defined as the sum of the central Earth angle ρ and the look angle θ,

$$\varphi = \theta + \arccos\left(1 - \frac{\Delta}{RE_C}\right) \tag{5.20}$$

Aside: Relationship of look angle to incidence angle

Using the fact that $\sin(180 - \varphi) = \sin\varphi$, and the law of sines, there is

$$\frac{\sin\theta}{RE_C} = \frac{\sin\varphi}{R_{SC}} \tag{5.21}$$

5.5 Scene Sizes and Natural Pixel Spacings

Scene width in range

The scene width in range D_R is the ground distance width of the image swath. If the image product is in ground range projection (PT = G), this is simply,

$$D_R = P \cdot \frac{PIXSIZ}{1000.0} \tag{5.22}$$

with the number of image pixels P and the pixel spacing PIXSIZ.

However, if the image is in a slant range projection, the ground range to the near and far portions of the image swath must first be determined. Then, their difference gives us the swath width (see Appendix B).

Scene length in azimuth

The scene length in azimuth D_A is the ground distance length of the image product. This is calculated the same for all product types as

$$D_A = L \cdot \frac{\text{LINESPACING}}{1000.0} \tag{5.23}$$

with the number of lines L and the line spacing LINESPACING.

Swath velocity

The swath velocity V_{SW} is the speed at which the swath is moving along the surface of the Earth at the beam center. Given the scene length in azimuth D_A, it is calculated simply as

$$V_{SW} = 1000.0 \cdot \frac{D_A}{\text{data interval} \cdot 2} \quad \text{(data interval taken from PPDR)} \tag{5.24}$$

Natural azimuth data spacing

The line spacing Δ_{AZ} in an image is determined by dividing swath velocity V_{SW} by the pulse repetition frequency (PRF). That is, the rate the swath is moving on the ground times the interval between pulses gives the distance between lines on the ground.

$$\Delta_{AZ} = \frac{V_{SW}}{\text{PRF}} \tag{5.25}$$

Natural (slant) range data spacing

The sample spacing Δ_{SR} in an image is determined by the speed of light C divided by the complex sampling rate f_S times 2. That is, the rate that the signal travels times the interval between sampling gives the distance between samples in the ground. The factor of 2 is included because of the round trip that each pulse/echo must make.

$$\Delta_{SR} = \frac{C}{2.0 \cdot f_s} \tag{5.26}$$

Aside:
The ground range pixel spacing Δ_{GR} can be calculated from the slant range spacing Δ_{SR} and the look angle θ using

$$\Delta_{GR} = \frac{\Delta_{SR}}{\sin\theta} \tag{5.27}$$

5.6 IMAGE HEADING FIELDS

Processed scene center heading

The scene center heading is the angle from North to the direction the spacecraft is traveling across the scene. The method for computing the heading is

1) calculate the geocentric latitudes for the beginning and ending spacecraft state vectors

$$\begin{aligned} \theta_0 &= \arccos\sqrt{\frac{x_0{}^2 + y_0{}^2}{x_0{}^2 + y_0{}^2 + z_0{}^2}} \\ \theta_2 &= \arccos\sqrt{\frac{x_2{}^2 + y_2{}^2}{x_2{}^2 + y_2{}^2 + z_2{}^2}} \end{aligned} \tag{5.28}$$

2) calculate the arc distance A, directly between the two latitudes at the radius of the first spacecraft position r

$$\begin{aligned} r &= \sqrt{x_0{}^2 + y_0{}^2 + z_0{}^2} \\ A &= r \cdot \left|\theta_2 - \theta_0\right| \end{aligned} \tag{5.29}$$

3) calculate the straight line distance d, between the beginning and ending spacecraft state vectors

$$d = \sqrt{(x_0 - x_2)^2 + (y_0 - y_2)^2 + (z_0 - z_2)^2} \tag{5.30}$$

4) the heading angle α, is taken to be the arcsine of the direct arc length, A, and the magnitude of the change in the satellite's position d

$$\alpha = \arcsin\left(\frac{A}{d}\right) \tag{5.31}$$

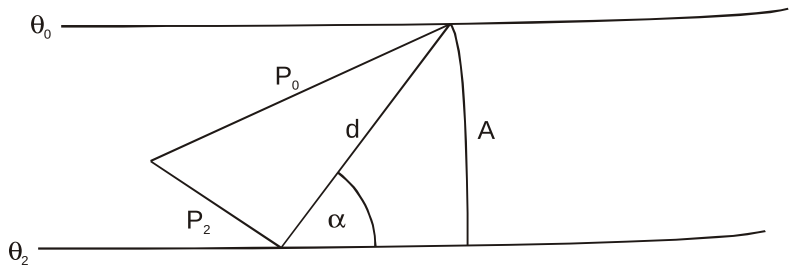

Figure 5.3: *Heading angle*

5) adjust the heading angle based on the direction of the satellite swath. For a descending swath, the angle α_1 (shown below) has been measured so that the actual heading from North is $\chi = 270 - \alpha_1$
   For an ascending swath, the angle α_2 (shown below) has been measured so that the actual heading from North is $\chi = 270 + \alpha_2$

Descending: $\chi = 270 - \alpha_2$ Ascending: $\chi = 270 + \alpha_2$

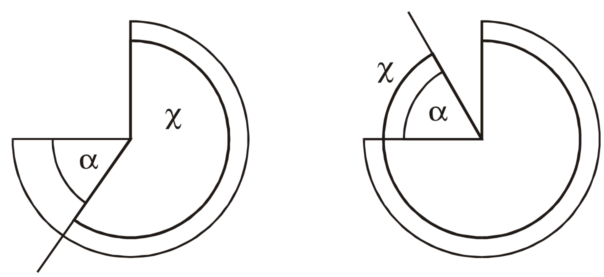

Figure 5.4: *Actual heading angle*

Platform heading / FACDR track angle

The platform heading is the angle from North to the direction the spacecraft is traveling across the scene. The method for computing the heading is

1) calculate the geocentric latitudes for the beginning and ending spacecraft state vectors

$$\theta_0 = \arccos\sqrt{\frac{x_0^2 + y_0^2}{x_0^2 + y_0^2 + z_0^2}}$$
$$\theta_2 = \arccos\sqrt{\frac{x_2^2 + y_2^2}{x_2^2 + y_2^2 + z_2^2}} \tag{5.32}$$

2) calculate the arc distance A, directly between the two latitudes at the radius of the Earth at the nadir position RE_N

$$A = RE_N \cdot \left|\theta_2 - \theta_0\right| \tag{5.33}$$

3) calculate the ground range length of the swath with the number of lines L and the pixel spacing PIXSIZ

$$d = L \cdot \frac{PIXSIZ}{1000.0} \tag{5.34}$$

4) the heading angle, α, is taken to be the arcsine of the direct arc length, A, and the ground range swath length, d

$$\alpha = \arcsin\left(\frac{A}{d}\right) \tag{5.35}$$

5) adjust the heading angle based on the direction of the satellite swath. For a descending swath, the angle α_1 (shown in Figure 5.4) has been measured so that the actual heading from North is $\chi = 270 - \alpha_1$
   For an ascending swath, the angle α_2 (shown in Figure 5.4) has been measured so that the actual heading from North is $\chi = 270 + \alpha_2$

   Descending: $\chi = 270 - \alpha_2$ Ascending: $\chi = 270 + \alpha_2$

   (see Figure 5.4)

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APPENDIX A: MATHEMATICAL BACKGROUND

I. Vector mathematics

magnitude of a vector, $\vec{V} = \{v_1, v_2, v_3\}$, is

$$|\vec{V}| = \sqrt{v_1{}^2 + v_2{}^2 + v_3{}^2} \qquad \text{(A.1)}$$

dot product of two vectors, $\vec{U}$ and $\vec{V}$, is

a. $\vec{U} \bullet \vec{V} \equiv u_1v_1 + u_2v_2 + u_3v_3$

b. $\vec{U} \bullet \vec{V} = |\vec{U}| \cdot |\vec{V}| \cdot \cos\theta$, where θ is the angle between $\vec{U}$ and $\vec{V}$ (A.2)

c. $\vec{U} \bullet \vec{V} = 0 \;\Leftrightarrow\; \vec{U} \perp \vec{V}$

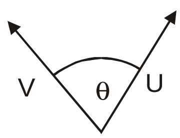


Figure C.1*: Dot product of two vectors*

cross product of two vectors, $\vec{U}$ and $\vec{V}$, is

a. $\vec{U} \times \vec{V} \equiv (u_1v_2 - u_2v_1, u_2v_0 - u_0v_2, u_0v_1 - u_1v_0)$

b. $\vec{U} \times \vec{V} \perp \vec{U}, \;\; \vec{U} \times \vec{V} \perp \vec{V}$

c. $|\vec{U} \times \vec{V}| = |\vec{U}| \cdot |\vec{V}| \cdot \sin\theta$, where θ is the angle between $\vec{U}$ and $\vec{V}$ (A.3)

d. $\vec{U} \times \vec{V} = 0 \;\Leftrightarrow\; \vec{U} \parallel \vec{V}$

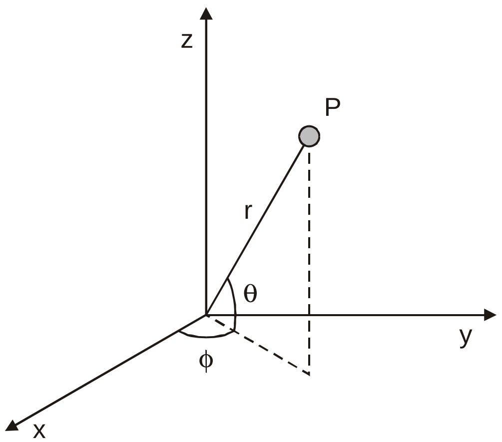
e. $\vec{U} \times \vec{V} = -(\vec{V} \times \vec{U})$

normalize – reduce magnitude of a vector to 1. Given $\vec{U}$,

$$\text{normalize}(\vec{U}) = \left\{ \frac{u_1}{|\vec{U}|}, \frac{u_2}{|\vec{U}|}, \frac{u_3}{|\vec{U}|} \right\} \tag{A.4}$$

II. Spherical coordinates

Spherical coordinates can be used to define a point in 3-D space much such as the standard cartesian coordinates. Spherical coordinates are characterized by two angles (ϕ, θ) and a radius r, as depicted in Figure C.2. Given a point P, with cartesian coordinates $\{x, y, z\}$, and spherical coordinates $\{r, \phi, \theta\}$, their relationships are



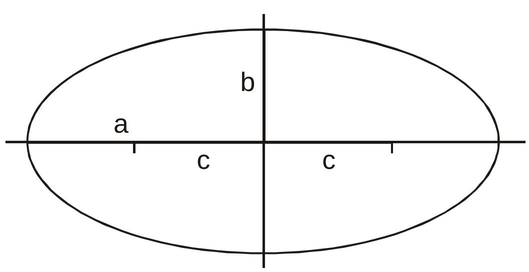
$$\begin{aligned} x &= r \cdot \cos\phi \cdot \cos\theta \\ y &= r \cdot \sin\phi \cdot \cos\theta \\ z &= r \cdot \sin\theta \end{aligned} \tag{A.5}$$

$$\begin{aligned} r &= \sqrt{x^2 + y^2 + z^2} \\ \phi &= \arctan\left(\frac{y}{x}\right) \\ \theta &= \arcsin\left(\frac{z}{r}\right) \end{aligned} \tag{A.6}$$

Figure C.2: *Spherical coordinates*

III. Definition of an ellipse

The general equation of an ellipse is



$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \quad a > b > 0 \tag{A.7}$$

Figure C.3: *Definition of an ellipse* for which the length of the major axis is 2a and the length of the minor axis is 2b. The definition of an ellipse is the set of all points P in a plane such that the sum of the distances from P to two other fixed points, called foci, is a positive constant. c is defined to be the distance from the center of the ellipse to the focus points. Then, $c = \sqrt{a^2 - b^2}$. Next, the eccentricity of an ellipse is defined to be the ratio of c to a,

$$\textit{eccentricity} = \frac{c}{a} = \frac{\sqrt{a^2 - b^2}}{a} \tag{A.8}$$

The relationship between the semi major axis a, the semi minor axis b, and the eccentricity e, can be summarized as follows:

$$a^2 = b^2 + c^2 \;\Rightarrow\; 1 = \frac{b^2}{a^2} + \frac{c^2}{a^2} \;\Rightarrow\; 1 = \frac{b^2}{a^2} + e^2 \;\Rightarrow\; b = a \cdot \sqrt{1 - e^2} \tag{A.9}$$

APPENDIX B: TRANSFORMATIONS FOR SLANT RANGE AND GROUND RANGE

I. Calculation of ground range from slant range

Given: R_{SC} = Range of spacecraft from center of Earth
SR_X = Slant Range to image pixel
RE_C = Earth radius at the center of the image

Calculate: GR_X = Ground range from nadir point to image pixel

Consider the imaging geometry depicted in Figure D.1. From the law of cosines:

$$SR_X^2 = RE_C^2 + R_{SC}^2 - 2 \cdot R_{SC} \cdot RE_C \cdot \cos\rho \tag{B.1}$$

So that for the central Earth angle ρ,

$$\cos\rho = \frac{RE_C^2 + R_{SC}^2 - SR_X^2}{2 \cdot R_{SC} \cdot RE_C} = \frac{1 + \frac{R_{SC}^2}{RE_C^2} - \frac{SR_X^2}{RE_C^2}}{2 \cdot R_{SC} \cdot RE_C} \tag{B.2}$$

Let:

$$a = \frac{R_{SC} - RE_C}{RE_C} = \frac{R_{SC}}{RE_C} - 1$$

$$x = 1 + a = \frac{R_{SC}}{RE_C} \tag{B.3}$$

$$y = \frac{SR_X}{RE_C}$$

Then:

$$\rho = \arccos\left(\frac{1 + x^2 - y^2}{2 \cdot x}\right) \tag{B.4}$$

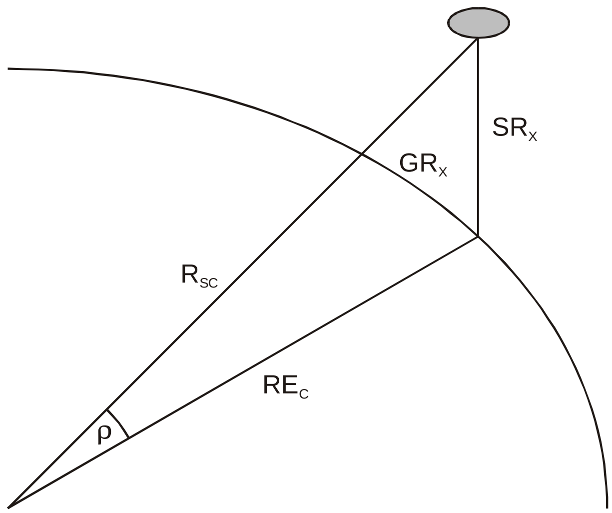


Figure B.1: *Range geometry*

Now, the ground range from the nadir point to the image pixel, GR_X, is $GR_X = \rho \cdot RE_C$,

So finally, $GR_X = RE_C \cdot \arccos\left(\frac{1 + x^2 - y^2}{2 \cdot x}\right)$ (B.5)

II. Calculation of slant range from ground range

Given: R_{SC} = Range of spacecraft from center of Earth
GR_X = Ground Range to image pixel
RE_C = Earth radius at the center of the image
ρ = center Earth angle

Calculate: SR_X = Slant range to image pixel

Consider the imaging geometry depicted above. From the law of cosines:

$$SR_X^2 = RE_C^2 + R_{SC}^2 - 2 \cdot R_{SC} \cdot RE_C \cdot \cos\rho \quad \text{(B.6)}$$

So that,

$$SR_X = RE_C \cdot \sqrt{1 + \frac{R_{SC}^2}{RE_C^2} - 2 \cdot \frac{R_{SC}}{RE_C} \cdot \cos\rho} \quad \text{(B.7)}$$

Rearranging, and adding and subtracting the quantity $2 \cdot \frac{R_{SC}}{RE_C}$ leads to

$$\begin{aligned} SR_X &= RE_C \cdot \sqrt{2 \cdot \frac{R_{SC}}{RE_C} - 2 \cdot \frac{R_{SC}}{RE_C} \cdot \cos\rho + \frac{R_{SC}^{\,2}}{RE_C^{\,2}} - 2 \cdot \frac{R_{SC}}{RE_C} + 1} \\ &= RE_C \cdot \sqrt{2 \cdot \frac{R_{SC}}{RE_C} \cdot (1 - \cos\rho) + \left(\frac{R_{SC}}{RE_C} - 1\right)^2} \end{aligned} \tag{B.8}$$

Substituting $\rho = \frac{GR_X}{RE_C}$,

$$SR_X = RE_C \cdot \sqrt{2 \cdot \frac{R_{SC}}{RE_C} \cdot \left[1 - \cos\left(\frac{GR_X}{RE_C}\right)\right] + \left(\frac{R_{SC}}{RE_C}\right)^2} \tag{B.9}$$

finally, substituting x and a into the equation,

$$SR_X = RE_C \cdot \sqrt{2 \cdot x \cdot (1 - \cos\left(\frac{GR_X}{RE_C}\right) + a^2} \tag{B.10}$$

III. Calculation of slant range to center pixel from ground range

Let G be the ground range from the near edge of the swath to the image center pixel.

Calculate GR_0, the ground range from the nadir to the start of the swath using the method in section I, where SR_X is replaced with the slant range to the first pixel.

Calculate GR_X, the ground range to the center of the image swath as GR_0 + G.

Calculate SR_X, the slant range to the center of the image swath using the method in section II.

IV. Calculation of swath width using slant ranges

Calculate rg0, the ground range to the near edge of the swath using the method in section I, where SR_X is replaced with the slant range to the first pixel, SR_F.

Calculate GR_1, the ground range to the far edge of the swath using the method in section I, where SR_X is replaced with the slant range to the last pixel, SR_L.

Then, the swath width in ground range is simply $GR_1 - GR_0$.

APPENDIX C: EARTH LATITUDES AND COORDINATE SYSTEMS

I. Geodetic and geocentric latitudes

Using an ellipsoid model complicates the calculation of the latitude for a point. Two commonly used definitions of latitude are displayed in Figure E.1. The first θ_C, is called the geocentric latitude and is defined to be the angle between the equatorial plane and the radius from the geocenter. The second θ_D, is the geodetic latitude and is defined to be the angle between the equatorial plane and the normal to the surface of the ellipsoid. In order to derive equations for the geodetic and geocentric latitudes, another type of latitude, the reduced latitude β, is defined. Using β, the coordinates for x can be written as

$$x = RE \cdot \cos\beta \tag{C.1}$$

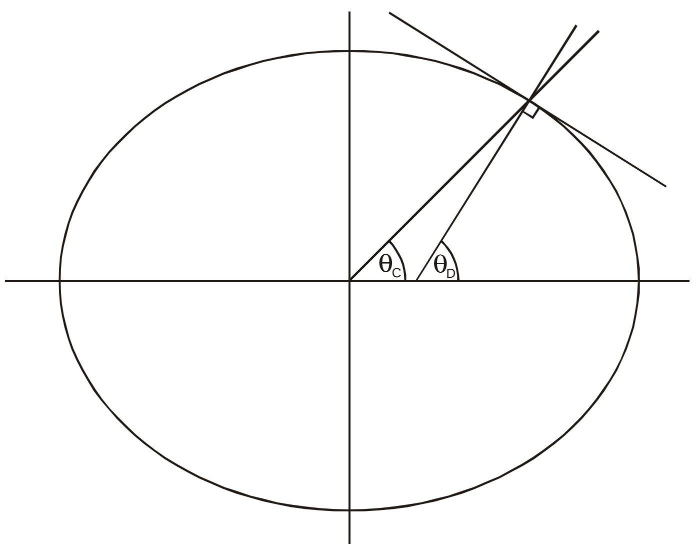


Figure C.1*: Geocentric and geodetic latitude*

In order to solve for the z coordinate on the ellipsoid, note that the x-coordinates are the same and use the definitions of an ellipse and a circle to establish the relationship between z', the point on the circumscribed circle, and z, the point on the ellipse. Given the equatorial Earth radius RE and the polar Earth radius RP, it is

$$\begin{aligned} &\frac{x^2}{RE^2} + \frac{z^2}{RP^2} = 1 \quad \text{[definition of an ellipse]} \\ &\Rightarrow \quad x^2 \cdot RP^2 + z^2 \cdot RE^2 = RE^2 \cdot RP^2 \end{aligned} \tag{C.2}$$

$$x^2 + z'^2 = RE^2 \quad \text{[definition of a circle]}$$
$$\Rightarrow \quad x^2 = RE^2 - z'^2 \tag{C.3}$$

substituting equation E.3 into equation E.2, leads to

$$(RE^2 - z'^2) \cdot RP^2 + z^2 \cdot RE^2 = RE^2 \cdot RP^2$$
$$(RE^2 \cdot RP^2 - z'^2 \cdot RP^2 + z^2 \cdot RE^2 = RE^2 \cdot RP^2$$
$$z^2 \cdot RE^2 = z'^2 \cdot RP^2 \tag{C.4}$$
$$z \cdot RE = z' \cdot RP$$
$$z = \frac{RP}{RE} \cdot z'$$

now, $z' = RE \cdot \sin\beta$, so finally,

$$z = \frac{RP}{RE} \cdot RE \cdot \sin\beta \tag{C.5}$$

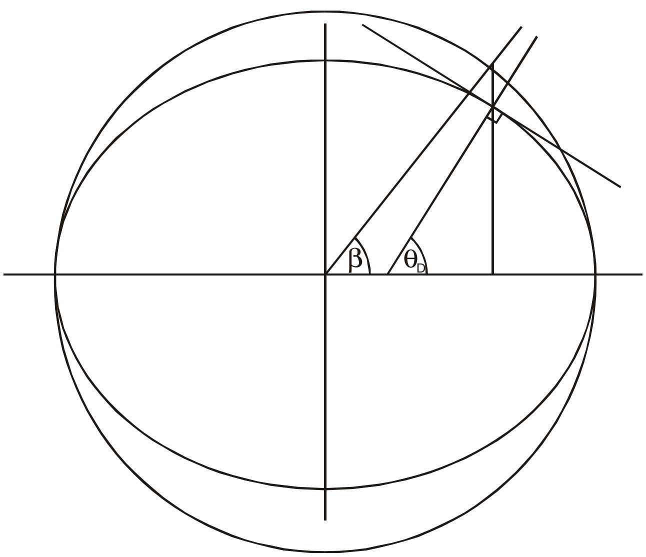


Figure C.2: *Reduced and geodetic latitude*

using the definition of the (squared) eccentricity of an ellipse ECC2, $RP = RE \cdot \sqrt{1 - ECC2}$, so that

$$z = RE \cdot \sqrt{1 - ECC2} \cdot \sin\beta \tag{C.6}$$

From calculus it is known that the slope of the tangent to the ellipse is $\frac{dz}{dx}$ and the slope of the normal is $-\frac{dx}{dz}$. Since the slope of the normal is just defined as tan θ_D

$$\tan\theta_D = -\frac{dx}{dz} \tag{C.7}$$

The differentials dx and dz are obtained by differentiating the expressions for x and z. Thus

$$\begin{aligned} dx &= -RE \cdot \sin\beta \cdot d\beta \\ dy &= RE \cdot \sqrt{1-ECC2} \cdot \cos\beta \cdot d\beta \end{aligned} \tag{C.8}$$

so that

$$\tan\theta_D = \frac{\tan\beta}{\sqrt{1-ECC2}}$$

or

$$\tan\beta = \sqrt{1-ECC2} \cdot \tan\theta_D = \sqrt{1-ECC2} \cdot \frac{\sin\theta_D}{\cos\theta_D} \tag{C.9}$$

this is the quotient,

$$\tan\beta = \frac{A}{B}, \text{ where } A = \sqrt{1-ECC2} \cdot \sin\theta_D \text{ and } B = \cos\theta_D \tag{C.10}$$

then,

$$\begin{aligned} \sin\beta &= \frac{A}{\sqrt{A^2+B^2}} = \frac{\sqrt{1-ECC2} \cdot \sin\theta_D}{\sqrt{1-e^2 \cdot \sin^2\theta_D}} \\ \cos\beta &= \frac{B}{\sqrt{A^2+B^2}} = \frac{\cos\theta_D}{\sqrt{1-e^2 \cdot \sin^2\theta_D}} \end{aligned} \tag{C.11}$$

finally then, the x and z coordinates for a point on the ellipse can be written as

$$\begin{aligned} x &= \frac{RE \cdot \cos\theta_D}{\sqrt{1-e2 \cdot \sin^2\theta_D}} \\ z &= \frac{RE \cdot \sqrt{1-ECC2} \cdot \sqrt{1-ECC2} \cdot \sin\theta_D}{\sqrt{1-e^2 \cdot \sin^2\theta_D}} \end{aligned} \tag{C.12}$$

Using similar methods, the equation for the y coordinate can be derived as well. The final conversion from geodetic coordinates to cartesian are thus:

$$\begin{aligned}
R &= \frac{RE}{\sqrt{1-(ECC \cdot \sin lat_d)^2}} \\
x &= R \cdot \cos lon \cdot \cos lat_d \\
y &= R \cdot \sin lon \cdot \cos lat_d \\
z &= R \cdot (1 - ECC2) \cdot \sin lat_d
\end{aligned} \tag{C.13}$$

The relationship between the geocentric and geodetic latitudes can be determined in the following way. Let z' be the coordinate on the sphere and z be the coordinate on the ellipse. It is given that $z = \frac{RP}{RE} \cdot z'$. Then,

$$\begin{aligned}
\tan\beta &= \frac{z'}{x} \;\Rightarrow\; z' = x \cdot \tan\beta \;\Rightarrow\; z = \frac{RP}{RE} \cdot x \cdot \tan\beta \\
\tan\theta_C &= \frac{z}{x} \;\Rightarrow\; z = x \cdot \tan\theta_C
\end{aligned} \tag{C.14}$$

thus,

$$\tan\theta_C = \frac{RP}{RE} \cdot \tan\beta \tag{C.15}$$

also,

$$\tan\beta = \sqrt{1 - ECC2} \cdot \tan\theta_D \tag{C.16}$$

so that,

$$\tan\theta_C = \frac{RP}{RE} \cdot \sqrt{1 - ECC2} \cdot \tan\theta_D \tag{C.17}$$

now, $\sqrt{1 - ECC2} = \frac{RP}{RE}$, that can be transformed to

$$\begin{aligned}
& \tan\theta_C = (1 - ECC2) \cdot \tan\theta_D \\
\text{or} \quad & \tan\theta_C = \left(\frac{RP}{RE}\right)^2 \cdot \tan\theta_D
\end{aligned} \tag{C.18}$$

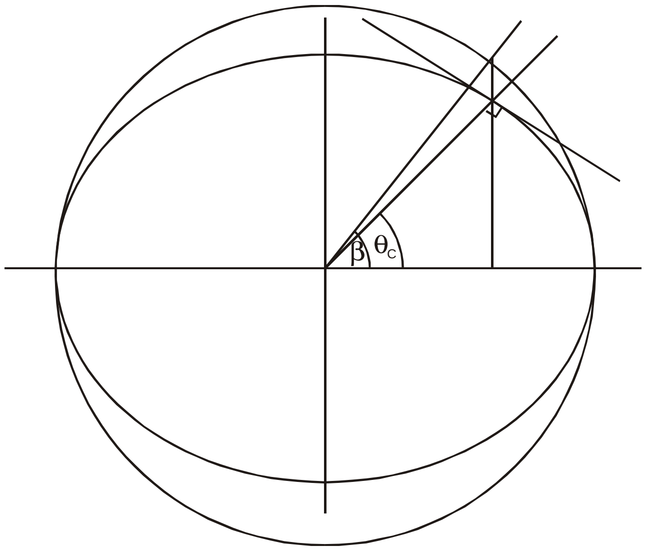


Figure C.3*: Reduced and geocentric latitude*

Finally then, given a cartesian vector, {x,y,z}, can be transformed into ellipsoidal coordinates with a geodetic latitude using the following:

$$\begin{aligned} r &= \mathrm{mag}\,(x, y, z) \\ \theta_C &= \arcsin\left(\frac{z}{r}\right) \\ \theta_D &= \arctan\left(\frac{\tan\theta_C}{1 - ECC2}\right) \\ \mathrm{lon} &= \arctan\left(\frac{y}{x}\right) \end{aligned} \tag{C.19}$$

II. Earth radius

Given a point P = {x,y,z}, the equatorial Earth radius RE and the polar Earth radius RP, where P lies on the surface of the Earth if the following holds true:

$$\frac{x^2 + y^2}{RE^2} + \frac{z^2}{RP^2} = 1 \tag{C.20}$$

Given a geocentric latitude θ_C, the radius of the Earth can be calculated by

$$\text{Radius} = \frac{RE \cdot RP}{\sqrt{(RP \cdot \cos\theta_C)^2 + (RE \cdot \sin\theta_C)^2}} \tag{C.21}$$

Given a geodetic latitude θ_D, the radius of the Earth can be calculated by

$$\begin{aligned} \theta_C &= \arctan(\tan\theta_d \cdot (1 - ECC2)) \\ \text{Radius} &= \frac{RE \cdot RP}{\sqrt{(RP \cdot \cos\theta_C)^2 + (RE \cdot \sin\theta_C)^2}} \end{aligned} \tag{C.22}$$

III. Earth coordinate systems

Earth Body Fixed Coordinates (EBF)

The EBF coordinate system is set up such that the x-axis passes through the equator at the Greenwich Meridian, the z-axis passes through the Earth's rotation axis, and the y-axis completes the right-hand rectangular system. Since the x-axis passes through the Greenwich meridian, this coordinate system rotates with the Earth. Thus, it is fixed to the body of the Earth. So, a given location on the surface of the Earth will always have the same location using EBF coordinates.

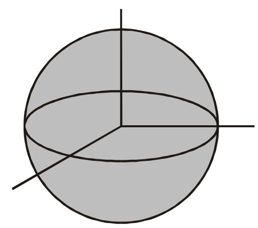


Figure C.4*:* *Earth coordinate systems*

Geocentric Equatorial Inertial Coordinates (GEI)

The GEI coordinate system, similar to EBF, has the z-axis passing through the Earth's rotation axis. However, the x-axis points in the direction of the vernal equinox, which is generally in the direction of the constellation Aries. The y-axis completes the right-hand system. Since the x-axis points in a constant direction, this coordinate system does not rotate with the Earth. This means a fixed point on the Earth will have continuously changing GEI coordinates with a velocity $\vec{V}_T$ of

$$\vec{V}_T = \vec{\omega}_E \times \vec{R}_T \tag{C.23}$$

where $\vec{\omega}_E$ is the vector $(0,0,\omega_E)$ and $\vec{R}_T$ is the GEI vector of the target point.

Coordinate Transformations

Transformation between these coordinate systems relies on knowing how far the Greenwich meridian is from the vernal equinox position at any given moment in time. This quantity, called the *Greenwich hour angle*, must be known in order to rotate from GEI to EBF or from EBF to GEI coordinates.

GEI to EBF:

Given a GEI state vector position, $\vec{P}_{GEI} = (x, y, z)$, a GEI state vector velocity, $\vec{V}_{GEI} = (vx, vy, vz)$, and the Greenwich hour angle, gha, in radians, the EBF coordinates, $\vec{P}_{EBF}$ and $\vec{V}_{EBF}$, can be calculated as follows:

<u>*Position:*</u>

Let

$$r = \sqrt{x^2 + y^2} \qquad \{\text{magnitude of x, y position}\}$$
$$lon = \arctan\left(\frac{y}{x}\right) \qquad \{\text{longitude of position}\} \tag{C.24}$$

Compute,

$$lon = lon - gha \qquad \{\text{subtract the gha from the longitude}\}$$
$$x' = \cos lon \cdot r \qquad \{\text{recompute x position}\}$$
$$y' = \sin lon \cdot r \qquad \{\text{recompute y position}\} \tag{C.25}$$
$$z' = z \qquad \{\text{z position does not change}\}$$

then,

$$\vec{P}_{EBF} = (x', y', z') \tag{C.26}$$

<u>*Velocity:*</u>

Let

$$r = \sqrt{vx^2 + vy^2} \qquad \{\text{magnitude of x, y velocity}\}$$
$$lon = \arctan\left(\frac{vy}{vx}\right) \qquad \{\text{longitude of velocity}\} \tag{C.27}$$

Compute,

$$lon = lon - gha \qquad \{\text{subtract the gha to the longitude}\}$$
$$vx' = \cos lon \cdot r + \omega_E \cdot y' \qquad \{\text{recompute x velocity including Earth spin}\}$$
$$vy' = \sin lon \cdot r - \omega_E \cdot x' \qquad \{\text{recompute y velocity including Earth spin}\} \tag{C.28}$$
$$vz' = vz \qquad \{\text{z velocity does not change}\}$$

then,

$$\vec{V}_{EBF} = (vx', vy', vz') \tag{C.29}$$

EBF to GEI:

Given an EBF state vector position, $\vec{P}_{EBF} = (x, y, z)$, an EBF state vector velocity, $\vec{V}_{EBF} = (vx, vy, vz)$, and the Greenwich hour angle, gha, in radians, the GEI coordinates, $\vec{P}_{GEI}$ and $\vec{V}_{GEI}$, can be calculated as follows:

<u>*Position:*</u>

Let

$$r = \sqrt{x^2 + y^2} \qquad \{\text{magnitude of x, y position}\}$$
$$lon = \arctan\left(\frac{y}{x}\right) \qquad \{\text{longitude of position}\} \tag{C.30}$$

Compute,

$$lon = lon + gha \qquad \{\text{add the gha from the longitude}\}$$
$$vx' = \cos lon \cdot r + \omega_E \cdot y' \qquad \{\text{recompute x position}\}$$
$$vy' = \sin lon \cdot r - \omega_E \cdot x' \qquad \{\text{recompute y position}\} \tag{C.31}$$
$$vz' = vz \qquad \{\text{z position does not change}\}$$

then,

$$\vec{P}_{GEI} = (x', y', z') \tag{C.32}$$

<u>*Velocity:*</u>

Let

$$r = \sqrt{vx^2 + vy^2} \qquad \{\text{magnitude of x, y velocity}\}$$
$$lon = \arctan\left(\frac{vy}{vx}\right) \qquad \{\text{longitude of velocity}\} \tag{C.33}$$

Compute,

$$lon = lon + gha \qquad \{\text{add the gha to the longitude}\}$$
$$vx' = \cos lon \cdot r - \omega_E \cdot y' \qquad \{\text{recompute x velocity including Earth spin}\}$$
$$vy' = \sin lon \cdot r + \omega_E \cdot x' \qquad \{\text{recompute y velocity including Earth spin}\} \tag{C.34}$$
$$vz' = vz \qquad \{\text{z velocity does not change}\}$$

then,

$$\vec{V}_{GEI} = (vx', vy', vz') \tag{C.35}$$

APPENDIX D: GEOLOCATION ALGORITHM

Given: actual Doppler frequency shift, fd_0
actual slant range, sr_0
spacecraft position vector, $\vec{R}_{SC}$
spacecraft velocity vector, $\vec{V}_{SC}$

Calculate: Target geolocation (lat, lon)
Look Angle, θ
Incidence Angle, φ
Yaw Angle, ϕ

Procedure:

$$fd = -999999$$
$$sr = -999999$$
$$\phi = 0.0$$
$$\theta = 20.0$$

While ($|fd - fd_0| > 0.01$ or $|sr - sr_0| > 0.01$)

{

calculate pointing vector, $\vec{P}$
calculate target position, $\vec{R}_T$
convert target position to lat, lon
calculate target velocity, $\vec{V}_T$
calculate incidence angle, φ
calculate Doppler frequency shift, fd

$$sr = \mathrm{mag}(\mathrm{diff}(\vec{R}_{SC}, \vec{R}_T))$$
$$vel = \mathrm{mag}(\mathrm{diff}(\vec{V}_{SC}, \vec{V}_T))$$
$$\theta = \theta + \arctan\left(\frac{sr_0 - sr}{\tan\varphi \cdot sr}\right)$$
$$\phi = \phi - \arcsin\left(\lambda \cdot \frac{fd_0 - fd}{2 \cdot vel}\right)$$

}

Notes: Curlander (1991) states, $f_D = 2 \cdot \frac{V_R \cdot \sin\phi}{\lambda}$, where V_R is the relative velocity of the target and platform. From this equation for the yaw angle ϕ, used above, can be determined.

To understand the update equation for θ, the following relation is used (let $\Delta sr = sr0 - sr$).

$$\tan\varphi = \frac{\Delta sr}{\Delta L} \Rightarrow \Delta L = \frac{\Delta sr}{\tan\varphi}$$
$$\tan\Delta\varphi = \frac{\Delta L}{sr} = \frac{\Delta sr}{\tan\varphi \cdot sr} \Rightarrow \Delta\theta = \arctan\left(\frac{\Delta sr}{\tan\varphi \cdot sr}\right) \tag{D.1}$$

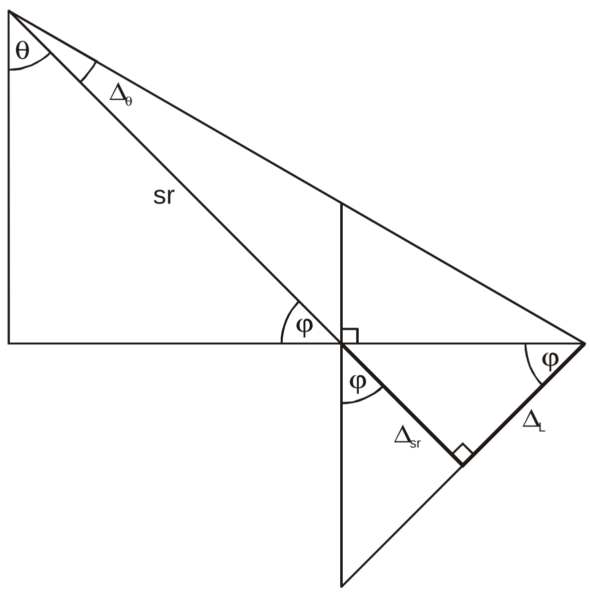


Figure D.1*: Look angle update (Δ_θ)*

I. Calculate pointing vector for the spacecraft

Given: Spacecraft Position Vector, $\vec{R}_{SC} = \{xs, ys, zs\}$
Spacecraft Velocity Vector, $\vec{V}_{SC} = \{xv, yv, zv\}$
Look Angle, θ
Yaw Angle, ϕ

Calculate: Pointing Vector, $\vec{P}$, for the Spacecraft

Let

$$\begin{aligned}\vec{u} &= \text{normalize}(\vec{V}_{SC} \times \vec{R}_{SC})\\ \vec{v} &= \text{normalize}(\vec{R}_{SC} \times \vec{u})\\ \vec{w} &= -1 \cdot \text{normalize}(\vec{R}_{SC})\end{aligned} \tag{D.2}$$

Then, $\vec{u}, \vec{v}, \vec{w}$ defines a coordinate system centered at the spacecraft, with $\vec{v}$ pointing in the direction of spacecraft flight, $\vec{w}$ pointing toward the center of the Earth, and $\vec{u}$ pointing directly to the right of the spacecraft. Using $\vec{u}, \vec{v}, \vec{w}$ as a basis, we can define a vector pointing at the look angle off of $\vec{w}$ and the yaw angle off of $\vec{u}$, by

$$\begin{aligned}P_x &= w_x \cdot \cos\theta + (u_x \cdot \cos\phi - v_x \cdot \sin\phi) \cdot \sin\theta\\ P_y &= w_x \cdot \cos\theta + (u_y \cdot \cos\phi - v_y \cdot \sin\phi) \cdot \sin\theta\\ P_z &= w_x \cdot \cos\theta + (u_z \cdot \cos\phi - v_z \cdot \sin\phi) \cdot \sin\theta\end{aligned} \tag{D.3}$$

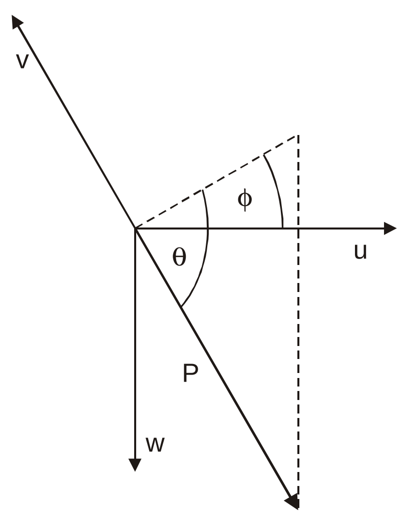


Figure D.2: *Point vector for the spacecraft*

II. Calculate target position

Given: Spacecraft Position Vector, $\vec{R}_{SC} = \{x_S, y_S, z_S\}$

Spacecraft Pointing Vector, $\vec{P} = \{x_P, y_P, z_P\}$

Calculate: Target Vector, $\vec{R}_T$, pointing from the center of the Earth to the exact surface

The vector $\vec{R} = \{x_T, y_T, z_T\}$ needs to be determined, such that $\vec{R}_T$ coincides with the surface of the Earth and $\vec{R}_T$ is a linear combination of the spacecraft vector and the pointing vector (i.e. $\vec{R}_T = \vec{R}_{SC} + \beta \cdot \vec{P}$) as depicted in Figure F.2. Using the second condition to define x_T, y_T, and z_T, and then plugging this into the first condition, leads to

$$\frac{(x_S + \beta \cdot x_P)^2 + (y_S + \beta \cdot y_P)^2}{RE^2} + \frac{(z_S + \beta \cdot z_P)^2}{RP^2} = 1 \qquad \text{(D.4)}$$

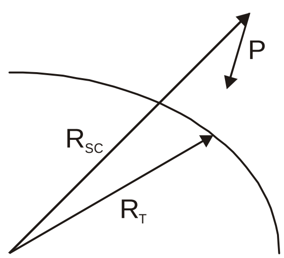


Figure D.3*: Target position vector*

expanding and rearranging

$$\underbrace{\left(\frac{x_P{}^2 + y_P{}^2}{RE^2} + \frac{z_P{}^2}{RP^2}\right)}_{A} \cdot \beta^2 + 2 \cdot \underbrace{\left(\frac{x_P \cdot x_S}{RE^2} + \frac{z_P \cdot z_S}{RP^2}\right)}_{B} \cdot \beta + \underbrace{\left(\frac{x_S{}^2 + y_S{}^2}{RE^2} + \frac{z_S{}^2}{RP^2} - 1\right)}_{C} = 0 \qquad \text{(D.5)}$$

We can now solve for β using the quadratic formula and calculate $\vec{R}_T$, the target vector

$$A = \frac{x_P{}^2 + y_P{}^2}{RE^2} + \frac{z_P{}^2}{RP^2}, \quad B = 2 \cdot \left(\frac{x_P \cdot x_S + y_P \cdot y_S}{RE^2} + \frac{z_P \cdot z_S}{RP^2}\right),$$
$$C = \frac{x_S{}^2 + y_S{}^2}{RE^2} + \frac{z_S{}^2}{RP^2} - 1 \qquad \text{(D.6)}$$

$$\beta = \frac{-B - \sqrt{B^2 - 4 \cdot A \cdot C}}{2 \cdot A} \qquad \text{(D.7)}$$

finally then,

$$\vec{R}_T = \vec{R}_{SC} + \beta \cdot \vec{P} \tag{D.8}$$

III. Conversion to lat, lon

Given: Target Vector

Calculate: Longitude, geocentric latitude, geodetic latitude

This is done exactly by the formulas developed for this in Appendix C.

IV. Calculate target velocity

Given: Target geodetic latitude and longitude

Calculate: Velocity of the target position, $\vec{V}_T$, due to Earth's spin

Let $\vec{R}_T = \{x_T, y_T, z_T\}$ be the cartesian coordinates for the target position (derived using the formulas developed to convert geodetic lat, lon to cartesian)

Note that $\sqrt{x_T^2 + y_T^2}$ is the radius from the z-axis (Earth's rotational axis) to the surface of the Earth. Then, $r_X = \omega_E \cdot \sqrt{x_T^2 + y_T^2}$ is the magnitude of the velocity of the Earth's spin at the target point, with the Earth rotation ω_E.

Let $\vec{V} = \text{normalize}(\vec{K} \times \vec{R}_T)$, where $\vec{K} = \{0,0,1\}$ is the unit vector in the z-direction. Then, $\vec{V}$, which is tangent to the surface of the Earth and perpendicular to the z-axis, points in the direction of the Earth's spin.

Then, $\vec{V}_T$, the target point's velocity is given by

$$\vec{V}_T = r_X \cdot \vec{V} \tag{D.9}$$

Note: $\vec{V}_T = \{0,0,\omega_E\} \times \vec{R}_T$ gives the same result as above

V. Calculate incidence angle

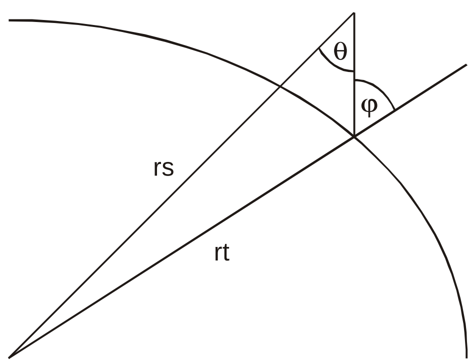
Given: Target Vector, $\vec{R}_T = \{x_T, y_T, z_T\}$

Spacecraft Vector, $\vec{R}_{SC} = \{x_S, y_S, z_S\}$
Look Angle, θ

Calculate: Incidence Angle, φ

$$r_S = \text{mag}(\vec{R}_T)$$
$$r_T = \text{mag}(\vec{R}_{SC})$$
$$\text{lat} = \arctan\left(\frac{z_T}{\sqrt{{x_T}^2 + {y_T}^2}}\right) \tag{D.10}$$
$$\text{lat\_d} = \arctan\left(\frac{\tan \text{lat}}{1 - \text{ECC2}}\right)$$
$$\text{dlat} = \text{lat\_d} - \text{lat}$$

$$\varphi = \arcsin\left(\frac{r_S}{r_T} \cdot \sin\theta\right) - \text{dlat} \tag{D.11}$$



$$\frac{\sin\varphi}{rs} = \frac{\sin\theta}{rt}$$

Figure D.4*: Incidence angle*

VI. Calculate Doppler frequency

Given: Spacecraft Position Vector, $\vec{R}_{SC}$
Spacecraft Velocity Vector, $\vec{V}_{SC}$
Target Position Vector, $\vec{R}_T$
Target Velocity Vector, $\vec{V}_T$

Calculate: Doppler frequency shift f_D, induced by relative velocities of spacecraft and target

Define,

$$\begin{aligned}\vec{R} &= \vec{R}_T - \vec{R}_{SC} \\ \vec{V}_R &= \vec{V}_T - \vec{V}_{SC} \\ R &= \left|\vec{R}\right|\end{aligned} \tag{D.12}$$

Then, the Doppler frequency shift is given by

$$f_D = \frac{-2.0}{\lambda \cdot R} \cdot (\vec{R} \bullet \vec{V}_R) \tag{D.13}$$

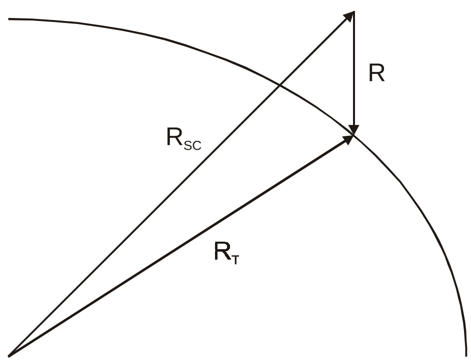


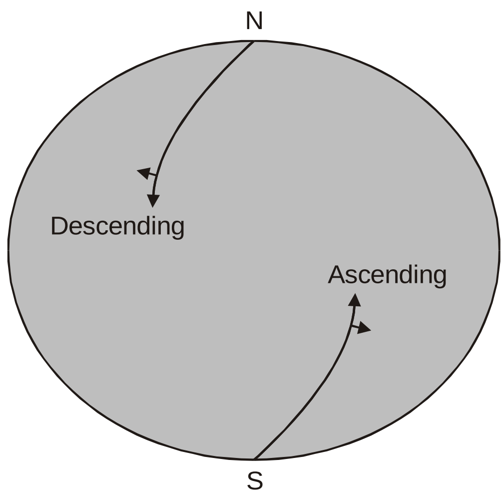
Figure D.5*: Geometry for Doppler frequency calculations*

(See Olmsted (1993), p. 19, for derivation of this equation for f_D)

Aside: For the relative acceleration from target to spacecraft we have, $\vec{A}_R = \vec{A}_T - \vec{A}_{SC}$.
Then the Doppler rate is given by $f_{DD} = \frac{-2.0}{\lambda \cdot R} \cdot \left|\vec{V}_R{}^2 - \vec{R} \bullet \vec{A}_R\right|$.

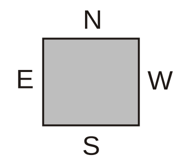
APPENDIX E: ASF IMAGE ORIENTATIONS

In normal imaging modes, ERS-1, ERS-2, JERS-1, and RadarSAT-1 are all right looking SAR platforms in polar orbits. The imaging orientations are thus fixed for ascending and descending scenes as depicted in the figure below.



Descending orbit:

Near swath	East
Far swath	West
Start swath	North
End swath	South



Ascending orbit:

Near swath	West
Far swath	East
Start swath	South
End swath	North

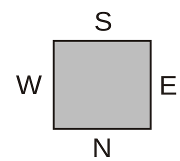
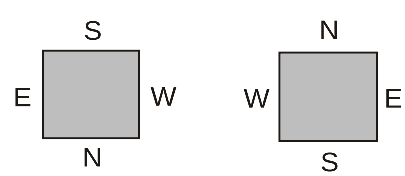


Figure G.1*: Ascending and descending orbits*

For convenience sake, ASF orients its image products such that only rotations are required when geocoding images. This requires some image manipulation. It turns out that a single flip in the azimuth direction is enough to satisfy this requirement. One can think of it as leaving the range direction's time as increasing, while reversing the azimuthal time direction to decreasing. In this case, image orientations as follows:

	Time direction	
	Descending	**Ascending**
Range	Increase	Increase
Azimuth	Decrease	Decrease

Table G.1: *Time direction*



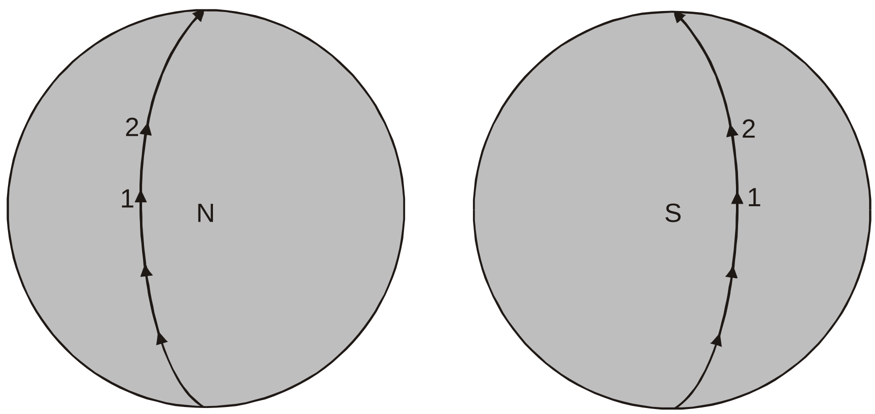


Figure G.2: *Time directions for ascending and descending orbits*

As an orbiting platform passes by the poles, the type of orbit (ascending or descending) changes, as do image orientations. Consider the cases above. For the South pole, as the platform passes from point 1 to point 2, the spacecraft heading goes from South to West to North. For the North Pole, it goes from North to West to South. Thus, images at the top and bottoms of orbits will have West at the start of the image frame in these areas.

INDEX