

## Height systems

Rüdiger Gens







#### Outline

- Why bother about height systems?
- Relevant terms
- Coordinate systems
- Reference surfaces
- Geopotential number
- Height systems





#### Why bother about height systems?

- give a meaning to a value defined for height
- combination of measurements from different sources
  - GPS measurements vs. leveling measurements
- three-dimensional calculations
  - SAR interferometry







#### **Relevant terms**

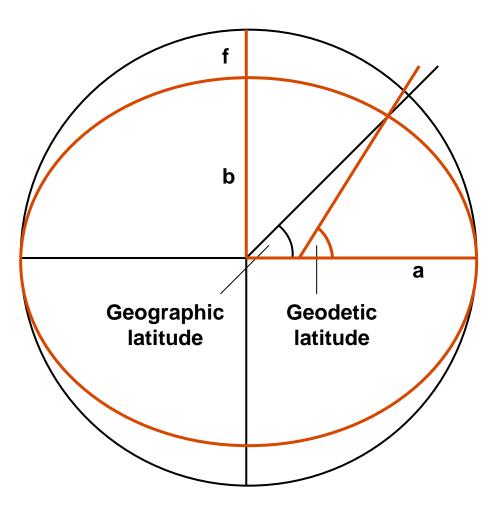
- spheriod
  - any surface resembling a sphere
  - an ellipsoid of revolution
- ellipsoid
  - defined by axes, flattening and eccentricity
- flattening and eccentricity
  - characterize the deviation from a sphere







## Geographical and geodetic coordinates









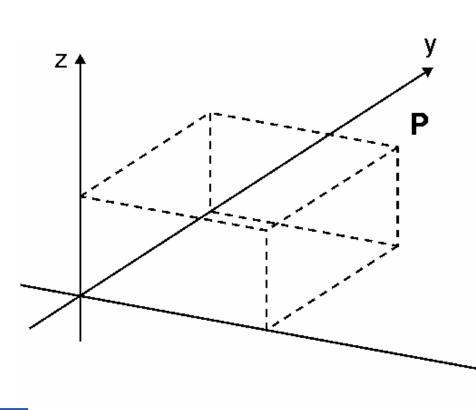
# Geographical and geodetic coordinates

- geographical coordinates
  - implying spherical Earth model
- geodetic coordinates
  - implying ellipsoidal Earth model





#### **Cartesian coordinates**



- geodetic coordinates inappropriate for satellite imagery
  - → cartesian coordinates

Х





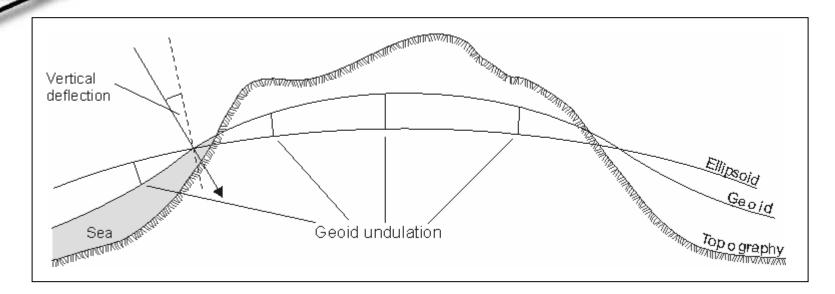


## **Approximation vs. Reality**

- ellipsoid is a good approximation to the shape of the Earth but not an exact representation
- Earth surface is everywhere perpendicular to the direction of gravity
   → equipotential surface
- true shape of the Earth is known as geoid



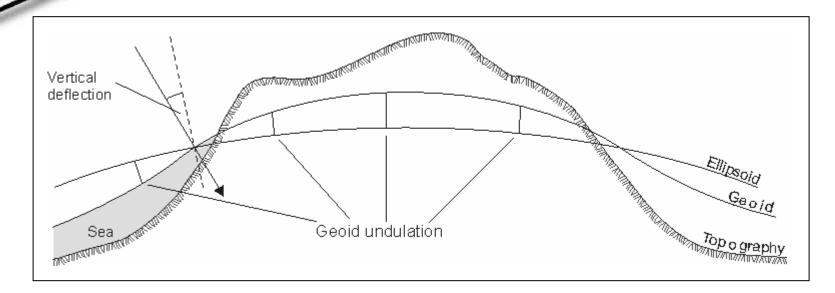




- three reference surfaces
  - topography
  - geoid
  - ellipsoid



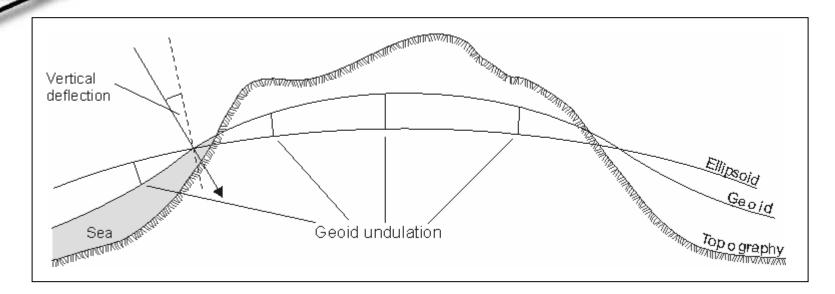




 topography represents the physical surface of the Earth



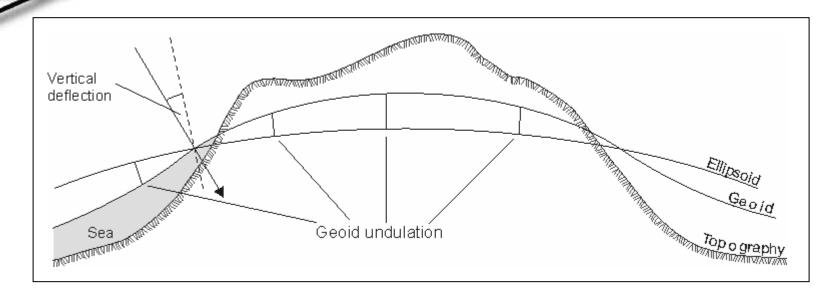




- geoid defined as level surface of gravity field with best fit to mean sea level
  - maximum difference between geoid and mean sea level about 1 m



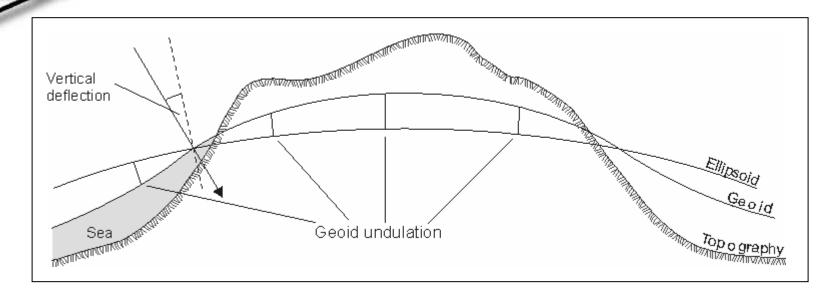




 ellipsoid defines mathematical surface approximating the physical reality while simplifying the geometry



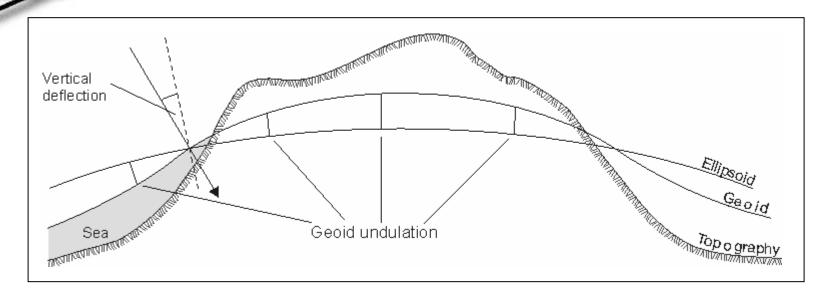




- geoid undulation: vertical separation between geoid and reference ellipsoid
  - differences between ± 100 m
  - global root mean square of around 30 m







- vertical deflection: angle between the ellipsoid normal and the plumb line
  - usually resolved in a north-south component  $\xi$  and an east-west component  $\eta$
  - angles usually amount to a few arc seconds





- geoid defined by a set of coefficients of a spherical harmonic expansion
  → global earth model
- several models available
  - OSU91
  - Earth Geopotential Model 1996 (EGM96)







 different height systems can be related to each other by the geopotential number C

$$\mathbf{C} = \mathbf{W}_0 - \mathbf{W} = \int_{\text{geoid}}^{\text{point}} \mathbf{g} \, \mathbf{dn}$$

- W and  $W_0$ : the potentials of gravity of a point and the geoid
- g: gravity value
- dn: leveling increment







#### **Geopotential number**

different heights calculated by dividing the geopotential number by a gravity value







## **Heights**

- dynamic height
  - constant normal gravity  $\gamma_0$  for an arbitrary standard latitude (usually 45 degrees)
  - no geometrical meaning
- orthometric height
  - natural "height above sea level"
  - measured along the current plumb line from the foot point on the geoid and the point on the surface
  - gravity value: mean gravity







## **Heights**

- normal height
  - vertical distance from terrain surface to the ellipsoid reduced by the height anomaly
  - measured along the ellipsoidal normal
  - gravity value: mean normal gravity







### Solution

- ellipsoid is convenient reference frame
  - mathematical figure
  - provides good approximation to the geoid
- geoid better height reference system
  - reference to mean sea level allows to use tide gauges as height reference points
  - physical significance: ensures horizontal representation of water surfaces such lakes and seas







#### Questions





