Height systems

Rüdiger Gens
Outline

• Why bother about height systems?
• Relevant terms
• Coordinate systems
• Reference surfaces
• Geopotential number
• Height systems
Why bother about height systems?

- give a meaning to a value defined for height
- combination of measurements from different sources
  - GPS measurements vs. leveling measurements
- three-dimensional calculations
  - SAR interferometry
Relevant terms

- **spheriod**
  - any surface resembling a sphere
  - an ellipsoid of revolution

- **ellipsoid**
  - defined by axes, flattening and eccentricity

- **flattening and eccentricity**
  - characterize the deviation from a sphere
Geographical and geodetic coordinates

Geographic latitude

Geodetic latitude

Height systems
Geographical and geodetic coordinates

- geographical coordinates
  - implying spherical Earth model

- geodetic coordinates
  - implying ellipsoidal Earth model
Cartesian coordinates

- geodetic coordinates inappropriate for satellite imagery
  $\rightarrow$ cartesian coordinates
Approximation vs. Reality

- ellipsoid is a good approximation to the shape of the Earth but not an exact representation
- Earth surface is everywhere perpendicular to the direction of gravity → *equipotential surface*
- true shape of the Earth is known as *geoid*
Reference surfaces

- three reference surfaces
  - topography
  - geoid
  - ellipsoid
• *topography* represents the physical surface of the Earth
• *geoid* defined as level surface of gravity field with best fit to mean sea level
  • maximum difference between geoid and mean sea level about 1 m
**ellipsoid** defines mathematical surface approximating the physical reality while simplifying the geometry
**Reference surfaces**

- **geoid undulation**: vertical separation between geoid and reference ellipsoid
  - differences between ± 100 m
  - global root mean square of around 30 m
Reference surfaces

- **vertical deflection**: angle between the ellipsoid normal and the plumb line
  - usually resolved in a north-south component $\xi$ and an east-west component $\eta$
  - angles usually amount to a few arc seconds
Global earth model

- geoid defined by a set of coefficients of a spherical harmonic expansion
  → global earth model

- several models available
  - OSU91
  - Earth Geopotential Model 1996 (EGM96)
Geopotential number

- different height systems can be related to each other by the geopotential number $C$

$$C = W_0 - W = \int_{\text{geoid}}^{\text{point}} g \, dn$$

- $W$ and $W_0$: the potentials of gravity of a point and the geoid
- $g$: gravity value
- $dn$: leveling increment
Geopotential number

- different heights calculated by dividing the geopotential number by a gravity value
Heights

- **dynamic height**
  - constant normal gravity $\gamma_0$ for an arbitrary standard latitude (usually 45 degrees)
  - no geometrical meaning

- **orthometric height**
  - natural “height above sea level”
  - measured along the current plumb line from the foot point on the geoid and the point on the surface
  - gravity value: mean gravity
Heights

- normal height
  - vertical distance from terrain surface to the ellipsoid reduced by the height anomaly
  - measured along the ellipsoidal normal
  - gravity value: mean normal gravity
Solution

- ellipsoid is convenient reference frame
  - mathematical figure
  - provides good approximation to the geoid

- geoid better height reference system
  - reference to mean sea level allows to use tide gauges as height reference points
  - physical significance: ensures horizontal representation of water surfaces such as lakes and seas
Questions